Towards Formal Planner Synthesis of Unified Legged and Armed Dynamic Locomotion in Constrained Environments

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1 Introduction

Contact-based decision and planning methods are increasingly being sought for task execution in humanoid robots [1]. Although widely used in the mobile robot motion planning community, formal verification and synthesis methods have not been yet incorporated into the motion planning for complex locomotion behaviors in humanoid robots. A possible reason is that humanoid robots are high dimensional and possess under-actuated dynamics. To circumvent this difficulty, our work in [2] proposed a phase-space planning strategy that leverages low-dimensional models for the locomotion process and still characterizes its essential dynamics. In particular, locomotion trajectories in the phase-space are sequentially composed based on determining keyframe states.

Given the aforementioned phase-space planner, this study takes a step toward formally synthesizing high-level reactive planners for unified legged and armed locomotion in constrained environments. We formulate a two-player temporal logic game between the contact planner and its possibly adversarial environment. The resulting discrete planner satisfies the given task specifications expressed in a fragment of temporal logic. The resulting commands are executed by a low-level 3D phase-space planner. We devise a set of specific low-level locomotion modes based on centroidal momentum dynamics. Provable correctness of the low-level execution of the synthesized high-level planner is guaranteed through the so-called simulation relations. Simulations of dynamic locomotion in constrained environments support the hierarchical planner protocol. We expect that this line of work acts as an entry point for the humanoid robotics community to employ formal methods to verify and synthesize motion planners.

2 Main Contribution

Our objective is to synthesize a correct high-level reactive planner for the unified locomotion problem. Fig. 1 shows an example scenario motivated by a naval research application: robot maneuvers and operates within a constrained environment such as submarine vessel. The main contribution of this work is to devise a high-level planner switching strategy by solving a two-player game. Environment actions are treated as adversaries in this game. We employ linear temporal logic (LTL) [3] to specify unified legged and armed locomotion (ULAL) behaviors. We focus on the communication between the high-level and low-level planners via switching signals and the correctness of the interplay. We rely on a discretization of the phase space into keyframe states. As an extension from rough terrain locomotion [2, 4], we focus not only on the ULAL behaviors but also on its response to unexpected environmental events such as stair crack and human appearance in the scene. To the best of our knowledge, this study is the first attempt to use formal methods for ULAL behaviors and planner synthesis with guarantee of correctness.

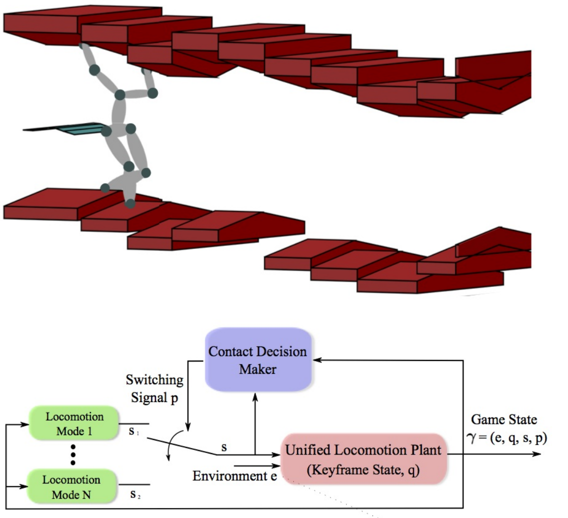

Figure 1: Maneuvering in a constrained environment via multi-contact legged and armed locomotion governed by a logic-based planner. Each mode, indexed by a switching signal p, corresponds to a locomotion model. The environment action is represented by e while the control action is represented by s describing the limb contact configuration. The discretized phase-space keyframe is $q = (p_{\text{contact}}, \dot{x}_{\text{apex}})$.

3 Switched Dynamics and Task Specifications

Dynamics of the unified armed and legged locomotion can be defined as a switched system with a switching signal p indexing specific locomotion modes. A logic-based switched system is shown in Fig. 1. Given a sequence of switching signals, the low-level planner evolves continuously in one mode and switches to the next one based on the computed contact transitions. To accomplish unified locomotion behaviors, we compose a sequence of locomotion modes with planned keyframes. This is achievable by synthesizing a high-level planner protocol which makes proper decisions on limb contacts and low-level planner switchings. Given these prelimi-

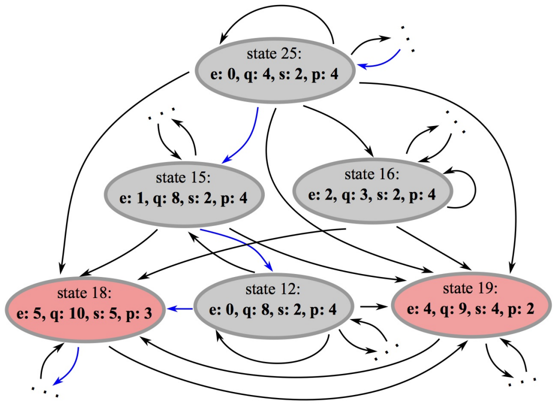

Figure 2: A fragment of the synthesized automaton for the ULAL contact planner. Nondeterministic transitions are encoded in this automaton. The blue transitions represent a specific execution. For convenience, the environment action e, keyframe state q, system action s and switching signal p are all indexed by numbers. For instance, when the state is at 25, we have $e = 0$ and $q = 4$. Then the winning strategy assigns $s = 2$ and $p = 4$.

naries, we formulate a discrete planner synthesis problem.

Discrete Contact Planner Switching Synthesis: Given a transition system $\mathcal{TS}$ and a LTL specification φ following an assume-guarantee form [3], we synthesize a contact planner switching strategy that generates only correct executions $\gamma = (q, p, e, s)$.

To make the computation tractable, we employ a class of LTL formulae with favorable polynomial complexity, named as the Generalized Reactivity (1) formulae [3]. Based on this formulae, we design LTL specifications involving environment actions e, system actions s and keyframe states q.

4 High-Level Reactive Planner Synthesis

Given a set of task specifications, we synthesize a reactive planner by formulating the high-level ULAL planning problem as a game between the robot and its environment. This ULAL planner game is a tuple $\mathcal{G}$, composed of input and output variables, initial states, transition relations and a LTL winning condition [3]. Given this game, a winning strategy of the switched system for the pair $(\mathcal{TS}, \varphi)$ is defined as a partial function $(\gamma_0\gamma_1 \cdots \gamma_{i-1}, (q_i, e_i)) \mapsto (s_i, p_i)$, where a contact configuration s_i and a switching mode p_i are chosen according to the state sequence history, the current keyframe state q_i and environment action e_i. All the specifications are satisfied whatever admissible yet uncontrollable environment actions are. *A winning ULAL strategy exists for the game $\mathcal{G}$ if and only if $(\mathcal{TS}, \varphi)$ is realizable.* Fig. 2 shows an automaton fragment of the ULAL contact planner. To guarantee the correctness of the hierarchical planning protocol, we define a mapping between the low-level trajectory and high-level execution, and conclude: *Given an over-approximation model, a winning ULAL strategy synthesized from the two-player game is guaranteed to be correctly implemented by the underlying low-level phase-space planners.*

5 Results

The locomotion tasks are achieved by combining the synthesized planner and the low-level planners via switched modes. The Temporal Logic Planning (TuLiP) toolbox, a python-based embedded control software [3], is used to synthesize the high-level contact planner. If the specifications are *realizable*, the synthesized planner is guaranteed by construction to satisfy all the specifications. Our resulting high-level planner is represented by a finite state automaton with 27 states and 148 transitions in total. For the low-level planners, four modes as shown in Fig. 3 are alternated according to the high-level switching protocol. Fig. 3 illustrates a synthesized CoM sagittal phase-space trajectory and snapshots of a 20-step dynamic walking. An accompanying video is available at https://youtu.be/urp7xu8vx3s.

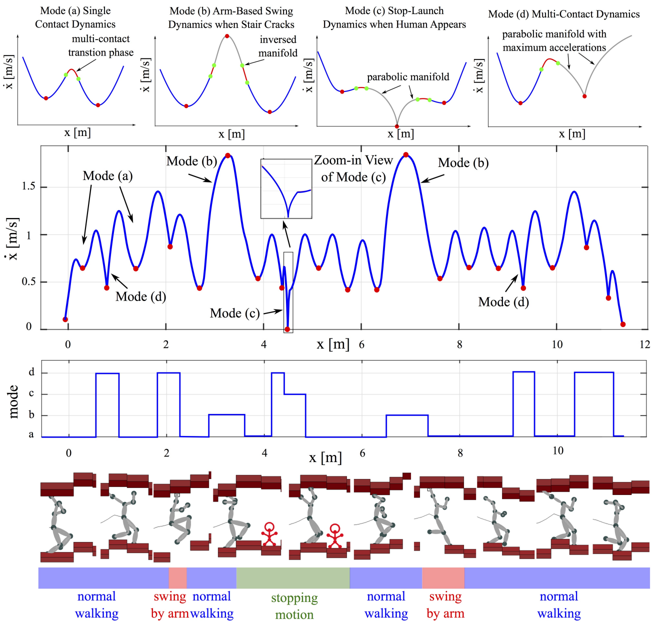

Figure 3: CoM sagittal phase-space trajectories, mode switchings and motion snapshots for a 20-step ULAL maneuver. The top four figures illustrate phase-space manifolds of different locomotion modes. The mode switching is governed by the high-level contact planner. Emergent behaviors are incorporated, i.e., two stair crack and one human appearance as shown in the locomotion snapshots at the bottom.

6 Ongoing Works

To enhance the applicability of the proposed synthesis method, we are studying more generalized locomotion tasks, such as locomotion with steered heading directions, in cluttered environments and along with humans. We also investigate practical problems including contact reachability and robot actuation limits. In the future, we plan to incorporate probabilistic models, such as Markov decision process or POMDP when the environment is partially observable.

References

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