Can Chance-Constrained Contact Uncertainty Quantification Improve Feasibility of Robust Trajectory Optimization?

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I. Summary

As robots move from the laboratory into the real world, motion planning and control will need to account for model uncertainty and risk. For robot motions involving intermittent contact, planning for uncertainty in contact is especially important, as failure to successfully make and maintain contact can be catastrophic. Here, we model uncertainty in terrain geometry and friction characteristics, and combine a risk-sensitive objective with chance constraints to provide a trade-off between robustness to uncertainty and constraint satisfaction with an arbitrarily high feasibility guarantee. We evaluate our approach in two simple examples: a push-block system and a single-legged hopper. We demonstrate that chance constraints alone produce trajectories similar to those produced using strict complementarity constraints; however, when equipped with a robust objective, we show the chance constraints can mediate a trade-off between robustness to uncertainty and strict constraint satisfaction (i.e. feasibility). Thus, our study may represent a step towards reasoning about contact uncertainty in motion planning.

II. Introduction

Designing safe and robust locomotion behaviors for legged robots is an important and challenging problem. Contact-implicit trajectory optimization recently gained attention for its ability to generate diverse locomotion behaviors [1]. However this method requires exact models of the terrain shape and friction characteristics which are extremely difficult to measure in real world environments. Modeling uncertainty has been approached in previous work by perturbing individual model parameters to generate an ensemble of reference trajectories [2]. In this work, we explicitly investigated uncertainty resulting from the terrain parameters and aimed to: (1) develop chance constraints for contact with uncertainty in contact distance and friction coefficient; (2) present a probabilistic interpretation to relaxed complementarity constraints using chance constraints; (3) demonstrate that chance constraints, combined with a contact-sensitive objective, can control the trade-off between robustness to uncertainty and constraint satisfaction.

III. Methods

Our study assumed normal distributions over the terrain parameters and developed a corresponding chance-constrained complementarity problem. By developing chance

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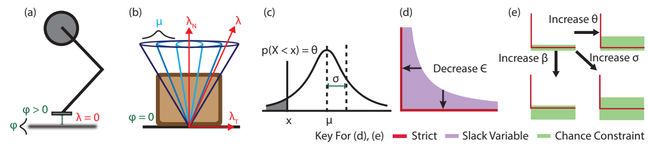

Fig. 1: (a),(b) Contact geometry of the hopper and block examples, respectively, with uncertainty in terrain height (hopper) and friction coefficient (block). (c) Gaussian distribution with mean μ and standard deviation σ, where $p(X < x) = \theta$. (d) Strict and slack variable relaxed complementarity constraints feasible regions. (e) Illustration of chance constraint feasible regions under different parameter changes.

constraints and treating a single contact parameter as normally distributed, we recovered a set of relaxed complementarity constraints. We also planned for terrain uncertainty with a previously developed risk-sensitive cost function [3]. We evaluated this approach on two toy examples: a sliding block with uncertain friction (Fig 1b) and a single legged hopper with uncertain terrain height(Fig 1a). We compared trajectories generated using various levels of terrain uncertainty to one generated with traditional contact-implicit approach.

IV. Results and Discussion

We characterized the feasibility regions of the chance complementarity constraints (Fig 1f). Chance constraints generalized the relaxed constraints in (Fig 1d [1]) and allowed for both force at a distance and terrain penetration. So far, we have shown that chance constraints, combined with a robust objective, mediate the trade-off between robustness and feasibility in the sliding block example. Our work represents the first time chance constraints have been incorporated in the contact model in contact-implicit trajectory optimization. We linked the relaxed complementarity problem [1] to probability theory, which could open up more robust problem modeling in trajectory optimization, as was done with state constraints [4]. In the future, we could focus on testing the robustness to variations in terrain parameters of stochastic MPC with chance constraints against traditional robust MPC approach without chance constraints.

References

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