Towards Data-Driven Contact Model Estimation using Inverse Optimization

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I. INTRODUCTION

Accurately identifying terrain shape and contact friction is a key challenge for robot locomotion especially as robots move from the lab to unstructured, real-world environments where contact models are intractable to provide *a priori*. While planning for uncertainty in contact could help to mitigate the risk of slipping or falling, locomotion systems should also be able to adapt their behaviors based on contact data gathered from experience; after a patch of uncertain rough terrain is traversed once, uncertainties in the terrain model should be reduced, making a second pass through easier to plan and execute. To date, there have been few attempts to update terrain models from experience. In this work, we propose an inverse optimization approach [1] to estimate contact reaction forces and contact model errors within the complementarity model of contact from given robot joint kinematics data. Our approach assumes only the executed controls and resulting state trajectories are available, and estimates both the contact forces and contact model errors. By inverting the complementarity constraints, we ensure that our estimated contact model is consistent with the analytical model used in trajectory optimization [2]. Moreover, our estimation approach also updates the uncertainty information in the contact model, which could be useful for contact-robust planning [3].

II. METHODS

A. Contact Estimation via Inverse Optimization

Our method uses the complementarity model of contact and develops an inverse optimization approach to estimate the contact distance and friction coefficient. We assume nominal models of contact distance and friction which are subject to linearly additive errors. Given state and control trajectory data, we formulate the inverse complementarity problem; that is, we solve for the reaction forces and the errors in the contact model which satisfy the robot dynamics and complementarity constraints. To ease the computational complexity, we relax the complementarity and dynamics constraints and include a cost on the feasibility of the solution, similar to the method used in [4]. Finally, we represent the contact distance and friction coefficient functions as Gaussian Processes (GPs), which allow us to effectively update the entire distance and friction functions using point-wise errors and to capture the uncertainty in the contact model.

B. Experiments

We evaluate our contact estimation algorithm in two simulated examples: a sliding block and a walking quadruped example. We first plan motion trajectories for each example using contact-implicit trajectory optimization [2], assuming a nominal flat ground model with constant friction. The optimal controls are then executed in simulation on different simulated terrains, including stepped terrain and terrain with non-constant friction. Using our proposed inverse optimization algorithm and the simulated joint kinematics, we estimate the reaction forces and the errors in the nominal contact model and update the terrain GPs using the estimated errors (Figure 1). Finally, we compare the updated terrain models to the models used in simulation to verify our approach.

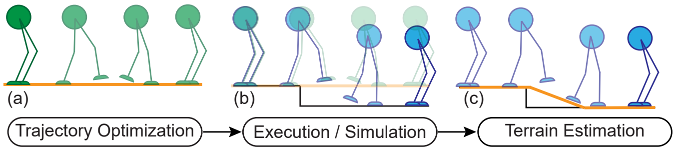

Fig. 1. Illustration of the relationship between contact-implicit planning and contact model estimation. (a) A locomotion trajectory is planned for a nominal flat-ground terrain model (orange). (b) The executed motion (blue) differs from the plan (green) because of an unmodeled disturbance arising from the true terrain (black). (c) The terrain model is updated to be consistent with the executed motion profile.

III. RESULTS AND DISCUSSION

We have found that our inverse optimization algorithm is accurately able to estimate errors in terrain height, even when such errors are large and would result in ground penetration under the nominal model. Similarly, our approach is able to reasonably recover spatial changes in the friction coefficient across the terrain, as well as the normal and frictional reaction forces. In all cases, the optimization at each time step solved within 10ms, indicating promise for real-time deployment.

In this work, we developed and tested a contact model estimation algorithm for updating contact models from data. By leveraging the complementarity framework for contact, we have ensured that our contact models are consistent with those used in contact-implicit trajectory optimization. Future works may combine our method with contact-robust optimization [3] for simultaneous planning and estimation and to adaptively replan motion trajectories as more online information is gained about the terrain.

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