Robust Stability Criterion for Discrete-Time Uncertain Markovian Jumping Neural Networks with Defective Statistics of Modes Transitions

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***Abstract*— This brief is concerned with the robust stability problem for a class of discrete-time uncertain Markovian jumping neural networks with defective statistics of modes transitions. The parameter uncertainties are considered to be norm-bounded, and the stochastic perturbations are described in terms of Brownian motion. *Defective statistics* means that the transition probabilities of the multimode neural networks are not exactly known, as assumed usually. The scenario is more practical, and such defective transition probabilities comprise three types: known, uncertain, and unknown. By invoking the property of the transition probability matrix and the convexity of uncertain domains, a sufficient stability criterion for the underlying system is derived. Furthermore, a monotonicity is observed concerning the maximum value of a given scalar, which bounds the stochastic perturbation that the system can tolerate as the level of the defectiveness varies. Numerical examples are given to verify the effectiveness of the developed results.**

***Index Terms*— Markovian jumping neural network, stability, transition probability matrix.**

I. Introduction

The past decades have witnessed extensive research on neural networks (NNs) in both mathematics and control communities, e.g., [1] and [2]. These studies are motivated by numerous applications of the NNs in diverse fields such as associative memory, pattern recognition, image processing, etc. As a major concern, the stability problem of the NNs has drawn much attention and a great number of efficient analysis approaches have been proposed in the literature, e.g., [3] and [4]. Meanwhile, considering the NNs involved with parameter uncertainties and/or stochastic perturbations, which frequently lead to the poor performance or even instability of the system, the corresponding stability analyses have also been widely investigated and many useful results have been obtained, e.g., [5] and [6] and the references therein.

Moreover, the NNs often display a feature of network modes jumpings and such jumpings are commonly considered to be determined by an ideal homogeneous Markov chain in the most literature. With the aid of analysis and synthesis methodologies in the dynamic systems with Markovian jumping parameters, i.e., the Markov jump linear systems (MJLSs), several significant results on the Markovian jumping neural networks (MJNNs) have been reported, e.g., [7] and [8]. It is worth mentioning that a recent interesting consideration for MJLSs is that the transition probabilities (TPs) to form the Markov chain are assumed to be not exactly known. The scenario containing such defective TPs is more general and the underlying MJLSs are thereby more practicable. Consequently, a few meaningful studies have been carried out, e.g., [9]–[12], and two concepts have been proposed so far, namely, the partially unknown TPs [11] and the uncertain TPs [10]. Also, the idea of the partially unknown TPs has recently been applied to the MJNNs [13].

For the concept of uncertain TPs, the elements in a transition probability matrix (TPM) are uncertain within an interval, and two description methods, namely the norm-bounded and the polytope uncertainty description, have been proposed. Correspondingly, the true elements in a TPM are unknown but belong to a *given* range with lower and upper bounds [10], or a *given* polytope with a certain number of vertices [14]. It should be noted that such *given* information is assumed obtainable when perfect statistics of the modes transitions is targeted in practical samplings and computations. On the other hand, the concept of partially unknown TPs assumes that some elements in a TPM are known, and others are not (even without any further *given* information of the statistics) [11]. Therefore, it can be well understood that the concept of partially unknown TPs is more general and the concept of uncertain TPs is less conservative since more information is “contrived” in the latter case.

In fact, the uncertain TPs can be considered as the unknown ones with further given knowledge offered from statistics. In reverse, the unknown TPs can also be viewed as uncertain ones within their “natural” intervals, which can be calculated from the known TPs and the property that the sum of each row is 1 in a TPM. In other words, the two concepts of the defective statistics are mathematically interrelated. Nevertheless, from a different viewpoint, such two concepts actually reflect different levels of the defectiveness. Note that, so far, these two lines of attacks of the defective statistics of modes transitions are still dealt separately. In fact, a more practical scenario that designers may encounter is that some TPs are known, some are uncertain with tighter intervals, and others are unknown with “natural intervals.” However, the issues on MJLSs taking account of the two aforesaid concepts of defective TPs in a composite manner are largely open, let alone the applications to the MJNNs area.

In this brief, we aim to address the robust stability problem for a class of discrete-time uncertain MJNNs with defective statistics of modes transitions. The parameter uncertainties are assumed to be norm-bounded, and the stochastic perturbations are described in terms of Brownian motion. The main contribution of this brief is that a framework incorporating the two concepts of the partially unknown TPs and the uncertain TPs is proposed for the first time and the underlying MJNN is studied under the framework. A sufficient stability criterion for the underlying system is obtained by using the property of

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the TPM and the convexity of uncertain domains. A monotonicity concerning the maximum value of a given scalar which bounds the stochastic perturbations affecting the system stability is observed as the level of the defectiveness varies. The remainder of this brief is organized as follows. In Section II, the mathematical model of the system concerned is formulated and some preliminary results are given. Section III is devoted to establishing the stability criterion for the underlying system and deriving several corollaries for the different simplified cases of the system. Numerical examples are provided in Section IV and this brief is concluded in Section V.

Notation: The notations used in this brief are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ refer to, respectively, the n-dimensional Euclidean space, and the set of all $m \times n$ real matrices, $\mathbb{N}^+$ stands for the sets of positive integers. The notation $P > 0$ (≥ 0) means P is real symmetric positive (semi-positive) definite and the superscript "T" denotes the transpose of vectors or matrices. Moreover, let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space, in which Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. In addition, in symmetric block matrices or long matrix expressions, we use * as an ellipsis for the terms that are introduced by symmetry and diag$\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. $E[\cdot]$ stands for the mathematical expectation and M_i is adopted to denote $M(i)$ for brevity. I and 0 represent, respectively, identity matrix and zero matrix with appropriate dimensions.

II. Preliminaries and Problem Formulation

Consider an n-neuron discrete-time uncertain Markovian jumping neural network, defined in a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$

$$\begin{aligned} y(k+1) &= (A(r(k)) + \Delta A(r(k)))y(k) + (B(r(k)) \\ &\quad + \Delta B(r(k))) \times f(y(k)) + \sigma(y(k), k)w(k) \end{aligned} \tag{1}$$

where $y(k) = (y_1(k), y_2(k), \ldots, y_n(k))^T \in \mathbb{R}^n$, is the state vector associated with the n neurons, $f(y(k)) = (f_1(y_1(k)), f_2(y_2(k)), \ldots, f_n(y_n(k)))^T \in \mathbb{R}^n$, denotes the nonlinear activation function with the initial condition $f(0) = 0$, $A(r(k)) = \text{diag}\{a_1(r(k)), a_2(r(k)), \ldots, a_n(r(k))\}$ has positive entries $a_m(r(k)) < 1$, $\forall m = 1, 2, \ldots, n$, the real matrix $B(r(k))$ is the constant connection weight matrix. In addition, $\Delta A(r(k))$, $\Delta B(r(k))$ are time-varying parameter uncertainties.

The Markov chain $\{r(k), k \in \mathbb{N}^+\}$ orchestrating the modes jumpings of the NNs takes values in a finite set $\mathcal{I} \triangleq \{1, \ldots, N\}$ with mode TPs $\Pr(r(k+1) = j | r(k) = i) = \pi_{ij}$, where $\pi_{ij} \geq 0$, $\forall i, j \in \mathcal{I}$, and $\sum_{j=1}^{N} \pi_{ij} = 1$. Correspondingly, the Markovian transition probability matrix Λ is defined by

$$\Lambda = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N} \\ & & \ddots & \\ \pi_{N1} & \pi_{N2} & \cdots & \pi_{NN} \end{bmatrix}. \tag{2}$$

The set $\mathcal{I}$ contains N modes of (1) and for $r(k) = i \in \mathcal{I}$, the system matrices of the i^{th} mode are denoted by $(A_i + \Delta A_i, B_i + \Delta B_i)$, which are real and known with compatible dimensions.

Taking account of the stochastic perturbations in forms of $\sigma(y(k), k)w(k)$, $w(k)$ is a scalar Brownian Motion on $(\Omega, \mathcal{F}, \mathcal{P})$ such that $E[w(k)] = 0$, $E[w^2(k)] = 1$.

Now we recall a necessary assumption for our derivation.

Assumption 1: The function $\sigma : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a Borel measurable function which satisfies

$$\sigma^T(x, k)\sigma(x, k) \leq \rho x^T x, \qquad \forall x \in \mathbb{R}^n \tag{3}$$

where ρ is a positive constant, which bounds the stochastic perturbations that the system can tolerate. More details of using ρ to describe the stochastic perturbations can be found in [15].

In addition, the parameters uncertainties, as commonly adopted in literature, e.g., [2] and [16], are assumed to have the structure $\forall r(k) = i$, $[\Delta A_i, \Delta B_i] = M_i F_i [N_{1i}, N_{2i}]$, where M_i, N_{1i} and N_{2i} are real constant matrices and F_i is an unknown time-varying matrix-valued function and satisfies $F_i^T F_i \leq I$, $\forall i \in \mathbb{N}^+$.

Remark 1: Note that, in practice, all the elements or a part of them in TPM (2) are probably costly or even impossible to obtain. Thus, instead of putting great efforts to measure or estimate the TPM, it is necessary and significant, from control perspectives, to further conduct research on the MJNNs with defective statistics of modes transitions.

In this brief, the statistics of modes transitions is considered to be defective. Specifically, some elements in matrix Λ are assumed not known exactly. They may be uncertain within given intervals offered from statistics, or they do not have such available intervals. We term the former as "uncertain" elements, and the latter as "unknown" ones. As described in [14], we assume that the TPM $\Lambda = [\pi_{ij}]_{N\times N}$ belongs to a given polytope $\mathbf{P}_\Lambda$ with vertices Λ_r, $r = 1, 2, \ldots, M$

$$\mathbf{P}_\Lambda \triangleq \left\{ \Lambda | \Lambda = \sum\nolimits_{r=1}^{M} \alpha_r \Lambda_r;\ \alpha_r \geq 0,\ \sum\nolimits_{r=1}^{M} \alpha_r = 1 \right\} \tag{4}$$

where $\Lambda_r = [\pi_{ij}]_{N\times N}$, $i, j \in \mathcal{I}$, $r = 1, \ldots, M$, are given TPMs containing unknown elements still. It is worth emphasizing that in (4), the property of each TPM Λ_r holds and the property of TPM Λ will be accordingly satisfied.

For simplicity, $\forall i \in \mathcal{I}$, we denote $\mathcal{I} = \mathcal{I}_{\mathcal{K}}^{(i)} \cup \mathcal{I}_{\mathcal{UC}}^{(i)} \cup \mathcal{I}_{\mathcal{UK}}^{(i)}$ as follows:

$$\mathcal{I}_{\mathcal{K}}^{(i)} \triangleq \{j : \pi_{ij} \text{ is known}\},\ \mathcal{I}_{\mathcal{UC}}^{(i)} \triangleq \{j : \tilde{\pi}_{ij} \text{ is uncertain}\}$$

and

$$\mathcal{I}_{\mathcal{UK}}^{(i)} \triangleq \{j : \hat{\pi}_{ij} \text{ is unknown}\}.$$

Here, each uncertain element and unknown element is labeled with the tide "˜" and "ˆ", respectively. Then, let $\pi_{\mathcal{UC}}^{(i)} \triangleq \sum_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\pi}_{ij}^{r}$, $\forall r = 1, \ldots, M$ and $\pi_{\mathcal{K}}^{(i)} \triangleq \sum_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \pi_{ij}$, respectively.

Remark 2: Note that the unknown elements actually have "natural intervals" which can be determined by the known elements, the lower and upper bounds of the uncertain elements, and the property that the sum of each row is 1 in a TPM. However, the reason of differentiating uncertain elements from unknown elements is that the uncertain elements with tighter intervals (not only the "natural intervals") reflect more information of the statistics and the TPM can be described more precisely.

The objective of this brief is to establish a stability criterion for (1) when the statistics of modes transitions is defective as stated in (4). To proceed further, we recall the essential assumption for the neuron activation function and the definition of asymptotic stability in the mean square for the underlying system.

Assumption 2: The neuron activation function in MJNN (1) is monotonically increasing and bounded, which satisfies

$$0 \leq \frac{f_j(s_1) - f_j(s_2)}{s_1 - s_2} \leq h, \qquad \forall j = 1, \ldots, n$$

where $s_1, s_2 \in \mathbb{R}$, $s_1 \neq s_2$, and h is a positive constant.

Definition 1: The MJNN (1) is said to be asymptotically stable in the mean square if, for any solution $y(k)$ of (1), the following holds:

$$\lim_{k \to \infty} E[| y(k) |^2] = 0.$$

III. Main Results

In this section, we will derive a stability criterion for the discrete-time uncertain MJNN (1) with defective statistics of modes transitions (4) and simplify the criterion when the complex dynamics in (1) are reduced. The following theorem presents a sufficient condition on the asymptotic stability in the mean square for (1).

Theorem 1: Consider the MJNN (1) with the defective TPM (4). Suppose that Assumptions 1 and 2 hold. The corresponding system is asymptotically stable in the mean square if there exist a set of matrices $P_i > 0$, a diagonal matrix $L > 0$, and positive scalars μ^* *and* ϵ, $\forall i \in \mathcal{I}$, such that

$$\mathcal{P}^i < \mu^* I \tag{5}$$

$$\Theta_i = \begin{bmatrix} -\mathcal{P}_s^i & \mathcal{P}_s^i A_i & \mathcal{P}_s^i B_i & \mathcal{P}_s^i M_i \\ * & \Gamma & L + \epsilon N_{1i}^T N_{2i} & 0 \\ * & * & \digamma & 0 \\ * & * & * & -\epsilon I \end{bmatrix} < 0 \tag{6}$$

where

$$\begin{cases} \mathcal{P}_{\mathcal{K}}^{(i)} \triangleq \sum_{j \in \mathcal{I}_{\mathcal{K}}^{(i)}} \pi_{ij} P_j, \ \mathcal{P}_{\mathcal{UC}}^{(i)} \triangleq \sum_{j \in \mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\pi}_{ij}^r P_j, \\ \mathcal{P}_{\mathcal{UK}}^{(i)} \triangleq \sum_{j \in \mathcal{I}_{\mathcal{UK}}^{(i)}} \hat{\pi}_{ij} P_j, \\ \mathcal{P}^i \triangleq \mathcal{P}_{\mathcal{K}}^{(i)} + \sum_{j \in \mathcal{I}_{\mathcal{UC}}^{(i)}} (\sum_{r=1}^M \alpha_r \tilde{\pi}_{ij}^r) P_j + \mathcal{P}_{\mathcal{UK}}^{(i)}, \\ \mathcal{P}_s^i \triangleq \mathcal{P}_{\mathcal{K}}^{(i)} + \mathcal{P}_{\mathcal{UC}}^{(i)} + (1 - \pi_{\mathcal{K}}^{(i)} - \pi_{\mathcal{UC}}^{(i)}) P_j, \ \forall j \in \mathcal{I}_{\mathcal{UK}}^{(i)} \end{cases} \tag{7}$$

and $\Gamma \triangleq \mu^* \rho I - P_i + \epsilon N_{1i}^T N_{1i}$, $\digamma \triangleq -2h^{-1} L + \epsilon N_{2i}^T N_{2i}$.

Proof: By Assumption 2 and $f(0) = 0$, it is straightforward to show that $0 \leq f(y_{ik})/(y_{ik}) \leq h$, when $s_2 = 0$. Since $f(y_{ik})$ is assumed to be monotonically increasing with the initial condition $f(0) = 0$, one knows $f(y_{ik}) > 0$ and $y_{ik} > 0$. Then, we can further show

$$y_{ik} - h^{-1} f(y_{ik}) \geq 0. \tag{8}$$

Multiplying (8) by $l_{ii} f(y_{ik})$ on the right, and since $l_{ii} > 0$, the above inequality is equivalent to $y_{ik} l_{ii} f(y_{ik}) - h^{-1} f(y_{ik}) l_{ii} f(y_{ik}) \geq 0$. By denoting a positive definite matrix $L \triangleq \text{diag}\{l_{11}, l_{22}, \ldots, l_{nn}\}$, $y_k \triangleq (y_{1k}, y_{2k}, \ldots, y_{nk})^T$ and $f(y_k) \triangleq (f(y_{1k}), f(y_{2k}), \ldots, f(y_{nk}))^T$, the following inequality holds:

$$y_k^T L f(y_k) - h^{-1} f^T(y_k) L f(y_k) \geq 0. \tag{9}$$

To derive the stability criterion, we introduce the following Lyapunov function candidate for (1), $V(y_k, k, r_k) = y_k^T P_i y_k$, $\forall r_k = i$, $i \in \mathcal{I}$. By (9), it follows that

$$\begin{aligned} E &\triangleq E[V(y_{k+1}, k+1, r_{k+1}) \,|\, y_k, r_k = i] - V(y_k, k, r_k) \\ &= y_{k+1}^T \mathcal{P}^i y_{k+1} - y_k^T P_i y_k \\ &= \left(\tilde{\mathcal{A}}_i y_k + \tilde{\mathcal{B}}_i f(y_k)\right)^T \mathcal{P}^i \left(\tilde{\mathcal{A}}_i y_k + \tilde{\mathcal{B}}_i f(y_k)\right) \\ &\quad + \sigma^T(y_k) \mathcal{P}^i \sigma(y_k) - y_k^T P_i y_k \\ &\leq \left(\tilde{\mathcal{A}}_i y_k + \tilde{\mathcal{B}}_i f(y_k)\right)^T \mathcal{P}^i \left(\tilde{\mathcal{A}}_i y_k + \tilde{\mathcal{B}}_i f(y_k)\right) + \sigma^T(y_k) \mathcal{P}^i \\ &\quad \times \sigma(y_k) - y_k^T P_i y_k + 2 y_k^T L f(y_k) - 2h^{-1} f^T(y_k) L f(y_k) \end{aligned}$$

where

$$\mathcal{P}^i \triangleq \sum_{j=1}^N \pi_{ij} P_j, \ \tilde{\mathcal{A}}_i \triangleq A_i + \Delta A_i, \ \tilde{\mathcal{B}}_i \triangleq B_i + \Delta B_i. \tag{10}$$

By Assumption 1 and (5), it can be readily shown that $\sigma^T(y_k) \mathcal{P}^i \sigma(y_k) \leq \mu^* \sigma^T(y_k) \sigma(y_k) \leq \mu^* \rho y_k^T y_k$, and then

$$\begin{aligned} E \ \leq\ & \left(\tilde{\mathcal{A}}_i y_k + \tilde{\mathcal{B}}_i f(y_k)\right)^T \mathcal{P}^i \left(\tilde{\mathcal{A}}_i y_k + \tilde{\mathcal{B}}_i f(y_k)\right) \\ & + y_k^T (\mu^* \rho I - P_i) y_k + 2 y_k^T L f(y_k) \\ & - 2h^{-1} f^T(y_k) L f(y_k). \end{aligned} \tag{11}$$

Further, we denote

$$\begin{cases} \Phi_i \triangleq \tilde{\mathcal{A}}_i y_k + \tilde{\mathcal{B}}_i f(y_k) \\ \Omega_i \triangleq y_k^T (\mu^* \rho I - P_i) y_k + 2 y_k^T L f(y_k) \\ \qquad -2h^{-1} f^T(y_k) L f(y_k). \end{cases} \tag{12}$$

Then, (11) becomes

$$E \leq \Phi_i^T \mathcal{P}^i \Phi_i + \Omega_i. \tag{13}$$

Now, we decompose the defective TPM considered in this brief

$$\begin{aligned} \mathcal{P}^i &= \sum_{j=1}^N \pi_{ij} P_j \\ &= \mathcal{P}_{\mathcal{K}}^{(i)} + \sum_{j \in \mathcal{I}_{\mathcal{UC}}^{(i)}} \left(\sum_{r=1}^M \alpha_r \tilde{\pi}_{ij}^r\right) P_j + \sum_{j \in \mathcal{I}_{\mathcal{UK}}^{(i)}} \hat{\pi}_{ij} P_j \end{aligned}$$

where $\sum_{r=1}^M \alpha_r \tilde{\pi}_{ij}^r$, $\forall j \in \mathcal{I}_{\mathcal{UC}}^{(i)}$ represents an uncertain element in the polytope uncertainty description. As $\sum_{r=1}^M \alpha_r = 1$ and α_r can take values arbitrarily in [0, 1], (13) implies that

$$\begin{aligned} E \ \leq\ & \Phi_i^T \left(\mathcal{P}_{\mathcal{K}}^{(i)} + \sum_{j \in \mathcal{I}_{\mathcal{UC}}^{(i)}} \left(\sum_{r=1}^M \alpha_r \tilde{\pi}_{ij}^r\right) P_j \right. \\ & \left. + \sum_{j \in \mathcal{I}_{\mathcal{UK}}^{(i)}} \hat{\pi}_{ij} P_j\right) \Phi_i + \Omega_i \\ =\ & \sum_{r=1}^M \alpha_r \left(\Phi_i^T \left(\mathcal{P}_{\mathcal{K}}^{(i)} + \sum_{j \in \mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\pi}_{ij}^r P_j \right.\right. \\ & \left.\left. + \sum_{j \in \mathcal{I}_{\mathcal{UK}}^{(i)}} \hat{\pi}_{ij} P_j\right) \Phi_i + \Omega_i\right). \end{aligned} \tag{14}$$

Then, (14) holds if and only if $\forall r = 1, \ldots, M$

$$\begin{aligned} E &\le \Phi_i^T \left(\mathcal{P}_{\mathcal{K}}^{(i)} + \mathcal{P}_{\mathcal{UC}}^{(i)} + \sum\nolimits_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}} \hat{\pi}_{ij} P_j\right)\Phi_i + \Omega_i \\ &= \Phi_i^T \Bigg(\mathcal{P}_{\mathcal{K}}^{(i)} + \mathcal{P}_{\mathcal{UC}}^{(i)} + \left(1 - \pi_{\mathcal{K}}^{(i)} - \pi_{\mathcal{UC}}^{(i)}\right) \\ &\qquad \times \sum\nolimits_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}} \frac{\hat{\pi}_{ij}}{1-\pi_{\mathcal{K}}^{(i)} - \pi_{\mathcal{UC}}^{(i)}} P_j\Bigg)\Phi_i + \Omega_i. \end{aligned} \quad (15)$$

Since

$$0 \le \frac{\hat{\pi}_{ij}}{1-\pi_{\mathcal{K}}^{(i)} - \pi_{\mathcal{UC}}^{(i)}} \le 1$$

and

$$\sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}} \frac{\hat{\pi}_{ij}}{1-\pi_{\mathcal{K}}^{(i)} - \pi_{\mathcal{UC}}^{(i)}} = 1$$

(15) becomes

$$\begin{aligned} E &\le \sum\nolimits_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}} \frac{\hat{\pi}_{ij}}{1-\pi_{\mathcal{K}}^{(i)} - \pi_{\mathcal{UC}}^{(i)}} \\ &\quad \times (\Phi_i^T(\mathcal{P}_{\mathcal{K}}^{(i)} + \mathcal{P}_{\mathcal{UC}}^{(i)} + (1-\pi_{\mathcal{K}}^{(i)} - \pi_{\mathcal{UC}}^{(i)})P_j)\Phi_i + \Omega_i). \end{aligned}$$

Thus, for $0 \le \hat{\pi}_{ij} \le 1 - \pi_{\mathcal{K}}^{(i)} - \pi_{\mathcal{UC}}^{(i)}$, the above inequality is equivalent to $\forall j \in \mathcal{I}_{\mathcal{UK}}^{(i)}$

$$E \le \Phi_i^T\left(\mathcal{P}_{\mathcal{K}}^{(i)} + \mathcal{P}_{\mathcal{UC}}^{(i)} + (1-\pi_{\mathcal{K}}^{(i)} - \pi_{\mathcal{UC}}^{(i)})P_j\right)\Phi_i + \Omega_i.$$

Considering (12) and $\mathcal{P}_s^i = \mathcal{P}_{\mathcal{K}}^{(i)} + \mathcal{P}_{\mathcal{UC}}^{(i)} + (1-\pi_{\mathcal{K}}^{(i)} - \pi_{\mathcal{UC}}^{(i)})P_j$, one knows that

$$\begin{aligned} E &\le \Phi_i^T \mathcal{P}_s^i \Phi_i + \Omega_i \\ &= \left(\tilde{\mathcal{A}}_i y_k + \tilde{\mathcal{B}}_i f(y_k)\right)^T \mathcal{P}_s^i \left(\tilde{\mathcal{A}}_i y_k + \tilde{\mathcal{B}}_i f(y_k)\right) \\ &\quad + y_k^T(\mu^*\rho I - P_i)y_k + 2y_k^T L f(y_k) - 2h^{-1} f^T(y_k) L f(y_k) \\ &= y_k^T(\tilde{\mathcal{A}}_i^T \mathcal{P}_s^i \tilde{\mathcal{A}}_i + \mu^*\rho I - P_i)y_k + f^T(y_k) \\ &\quad \times (\tilde{\mathcal{B}}_i^T \mathcal{P}_s^i \tilde{\mathcal{B}}_i - 2h^{-1}L) f(y_k) + 2y_k^T(\tilde{\mathcal{A}}_i^T \mathcal{P}_s^i \tilde{\mathcal{B}}_i + L) f(y_k) \\ &= \xi_k^T \tilde{\Pi} \xi_k \end{aligned} \quad (16)$$

where

$$\xi_k \triangleq \left[y_k^T \quad f^T(y_k)\right]^T$$

$$\tilde{\Pi} \triangleq \begin{bmatrix} \tilde{\mathcal{A}}_i^T \mathcal{P}_s^i \tilde{\mathcal{A}}_i + \mu^*\rho I - P_i & \tilde{\mathcal{A}}_i^T \mathcal{P}_s^i \tilde{\mathcal{B}}_i + L \\ * & \tilde{\mathcal{B}}_i^T \mathcal{P}_s^i \tilde{\mathcal{B}}_i - 2h^{-1}L \end{bmatrix}.$$

By Schur complement, (6) implies that $\forall i \in \mathcal{I}$

$$\begin{bmatrix} -\mathcal{P}_s^i & \mathcal{P}_s^i A_i & \mathcal{P}_s^i B_i \\ * & \Gamma & L + \epsilon N_{1i}^T N_{2i} \\ * & * & \digamma \end{bmatrix} + \epsilon^{-1} \tilde{\mathcal{P}}_m^i M_i M_i^T \tilde{\mathcal{P}}_m^{iT} < 0 \quad (17)$$

where $\tilde{\mathcal{P}}_m^i \triangleq [\mathcal{P}_s^{iT}, 0, 0]^T$. Meanwhile, we denote

$$\Upsilon \triangleq [0, \Delta A_i, \Delta B_i], \; \tilde{N} \triangleq [0, N_{1i}, N_{2i}]$$

$$\Pi \triangleq \begin{bmatrix} -\mathcal{P}_s^i & \mathcal{P}_s^i A_i & \mathcal{P}_s^i B_i \\ * & \mu^*\rho I - P_i & L \\ * & * & -2h^{-1}L \end{bmatrix}$$

$$\Delta\Pi \triangleq \tilde{\mathcal{P}}_m^i \Upsilon + \Upsilon^T \tilde{\mathcal{P}}_m^{iT}.$$

Thus, by [6, Lemma 1], we can verify that

$$\begin{aligned} \Delta\Pi &= \tilde{\mathcal{P}}_m^i \Upsilon + \Upsilon^T \tilde{\mathcal{P}}_m^{iT} \\ &= \tilde{\mathcal{P}}_m^i M_i F_i \tilde{N} + \tilde{N}^T F_i^T M_i^T \tilde{\mathcal{P}}_m^{iT} \\ &\le \epsilon \tilde{N}^T \tilde{N} + \epsilon^{-1} \tilde{\mathcal{P}}_m^i M_i M_i^T \tilde{\mathcal{P}}_m^{iT}. \end{aligned} \quad (18)$$

Then it follows from (17) and (18) that

$$\begin{aligned} \Pi + \Delta\Pi &= \begin{bmatrix} -\mathcal{P}_s^i & \mathcal{P}_s^i A_i & \mathcal{P}_s^i B_i \\ * & \mu^*\rho I - P_i & L \\ * & * & -2h^{-1}L \end{bmatrix} \\ &\quad + \tilde{\mathcal{P}}_m^i \Upsilon + \Upsilon^T \tilde{\mathcal{P}}_m^{iT} \\ &\le \begin{bmatrix} -\mathcal{P}_s^i & \mathcal{P}_s^i A_i & \mathcal{P}_s^i B_i \\ * & \mu^*\rho I - P_i & L \\ * & * & -2h^{-1}L \end{bmatrix} \\ &\quad + \epsilon \tilde{N}^T \tilde{N} + \epsilon^{-1} \tilde{\mathcal{P}}_m^i M_i M_i^T \tilde{\mathcal{P}}_m^{iT} \\ &= \begin{bmatrix} -\mathcal{P}_s^i & \mathcal{P}_s^i A_i & \mathcal{P}_s^i B_i \\ * & \Gamma & L + \epsilon N_{1i}^T N_{2i} \\ * & * & \digamma \end{bmatrix} \\ &\quad + \epsilon^{-1} \tilde{\mathcal{P}}_m^i M_i M_i^T \tilde{\mathcal{P}}_m^{iT} < 0. \end{aligned}$$

By (10), we have

$$\Pi + \Delta\Pi = \begin{bmatrix} -\mathcal{P}_s^i & \mathcal{P}_s^i \tilde{\mathcal{A}}_i & \mathcal{P}_s^i \tilde{\mathcal{B}}_i \\ * & \mu^*\rho I - P_i & L \\ * & * & -2h^{-1}L \end{bmatrix} < 0$$

which, by Schur complement, implies that

$$\tilde{\Pi} = \begin{bmatrix} \tilde{\mathcal{A}}_i^T \mathcal{P}_s^i \tilde{\mathcal{A}}_i + \mu^*\rho I - P_i & \tilde{\mathcal{A}}_i^T \mathcal{P}_s^i \tilde{\mathcal{B}}_i + L \\ * & \tilde{\mathcal{B}}_i^T \mathcal{P}_s^i \tilde{\mathcal{B}}_i - 2h^{-1}L \end{bmatrix} < 0. \quad (19)$$

From (16) and (19), for a negative scalar δ, we know

$$E = E[V(y_{k+1}, k+1, r_{k+1}) \,|\, y_k, r_k = i] - V(y_k, k, r_k) \le \delta\, |\xi_k|^2$$

which is equal to

$$E[V(y_{k+1}, k+1, r_{k+1})] - E[V(y_k, k, r_k)] \le \delta E[|\,\xi_k\,|^2]. \quad (20)$$

Given a positive integer m, the recursive sum of both sides of (20) from zero to m implies

$$E[V(y_{m+1}, m+1, r_{m+1})] - E[V(y_0, 0, r_0)] \le \delta \sum\nolimits_{k=0}^{m} E[|\xi_k|^2]$$

which gives $-\delta \sum_{k=0}^{m} E[|\,\xi_k\,|^2] \le E[V(y_0, 0, r_0)]$. Letting $m \to +\infty$, we know that the series $\sum_{k=0}^{m} E[|\,\xi_k\,|^2]$ is convergent, which means $\lim_{k\to+\infty} E[|\,y_k\,|^2] = 0$, hence the proof is completed. □

Remark 3: Note that the MJNN treated in Theorem 1 covers two simplified cases, i.e., the MJNN only with parameter uncertainties or only with stochastic perturbations, which we will address as follows. The proofs of the corresponding corollaries can be obtained in the same vein as the proof for Theorem 1.

Case 1: If there are no parameter uncertainties $\Delta A_i, \Delta B_i$ in the MJNN, the system reduces to

$$y(k+1) = A_i y(k) + B_i f(y(k)) + \sigma(y(k), k)\omega(k) \quad (21)$$

where the system matrices (A_i, B_i) are the same as the ones in (1). Then, we have the following corollary.

Corollary 1: Consider the MJNN (21) with the defective TPM (4). Suppose that Assumptions 1 and 2 hold. The corresponding system is asymptotically stable in the mean square if there exist a set of matrices $P_i > 0$, a diagonal matrix $L > 0$, and positive scalar μ^*, $\forall i \in \mathcal{I}$, such that $\mathcal{P}^i < \mu^* I$ and

$$\Theta_i = \begin{bmatrix} -\mathcal{P}_s^i & \mathcal{P}_s^i A_i & \mathcal{P}_s^i B_i \\ * & \mu^*\rho I - P_i & L \\ * & * & -2h^{-1}L \end{bmatrix} < 0$$

where the parameters $\mathcal{P}^i$ and $\mathcal{P}_s^i$ are the same as those in (7).

Case 2: If there are no stochastic perturbations $\sigma(y(k), k)$ $w(k)$ in the MJNN, the system reduces to

$$y(k+1) = (A_i + \Delta A_i)y(k) + (B_i + \Delta B_i)f(y(k)) \quad (22)$$

where the system matrices $(A_i + \Delta A_i,\ B_i + \Delta B_i)$ are the same as those in (1). Then, we have the following corollary.

Corollary 2: Consider the MJNN (22) with the defective TPM (4). Suppose that Assumptions 1 and 2 hold. The corresponding system is asymptotically stable in the mean square if there exist a set of matrices $P_i > 0$, a diagonal matrix $L > 0$, and positive scalars μ^* and ϵ, $\forall i \in \mathcal{I}$, such that $\mathcal{P}^i < \mu^* I$ and

$$\Theta_i = \begin{bmatrix} -\mathcal{P}_s^i & \mathcal{P}_s^i A_i & \mathcal{P}_s^i B_i & \mathcal{P}_s^i M_i \\ * & -P_i + \epsilon N_{1i}^T N_{1i} & L + \epsilon N_{1i}^T N_{2i} & 0 \\ * & * & F & 0 \\ * & * & * & -\epsilon I \end{bmatrix} < 0$$

where the parameters $\mathcal{P}^i$ and $\mathcal{P}_s^i$ are the same as those in (7).

Remark 4: Note also that the elements of the defective TPM in Theorem 1, which include the three sorts of TPs, i.e., known, uncertain, and unknown, could reduce to their different simplified cases shown as below (two sorts or one sort). Correspondingly, the composition of the parameters $\mathcal{P}^i$ and $\mathcal{P}_s^i$ in (7) will be different.

1) All the elements in the TPM are unknown. The corresponding system can be considered as the so-called switched NN under arbitrary switching, in terms of the analyses in [11]. Then we have
$$\mathcal{P}^i = \mathcal{P}_{\mathcal{UK}}^{(i)},\ \mathcal{P}_s^i = P_j,\ \forall j \in \mathcal{I}_{\mathcal{UK}}^{(i)}.$$
2) The TPM only contains known and unknown elements [9], and we have
$$\mathcal{P}^i = \mathcal{P}_{\mathcal{K}}^{(i)} + \sum\nolimits_{j \in \mathcal{I}_{\mathcal{UK}}^{(i)}} \hat{\pi}_{ij} P_j$$
$$\mathcal{P}_s^i = \mathcal{P}_{\mathcal{K}}^{(i)} + \left(1 - \pi_{\mathcal{K}}^{(i)}\right) P_j,\ \forall j \in \mathcal{I}_{\mathcal{UK}}^{(i)}.$$
3) The TPM only contains known and uncertain elements [10], then we have
$$\mathcal{P}^i = \mathcal{P}_{\mathcal{K}}^{(i)} + \sum\nolimits_{j \in \mathcal{I}_{\mathcal{UC}}^{(i)}} \left(\sum\nolimits_{r=1}^{M} \alpha_r \tilde{\pi}_{ij}^r\right) P_j$$
$$\mathcal{P}_s^i = \mathcal{P}_{\mathcal{K}}^{(i)} + \mathcal{P}_{\mathcal{UC}}^{(i)}.$$
4) All the elements are known. The corresponding system becomes the conventional MJNN with completely known TPM [17]
$$\mathcal{P}^i = \mathcal{P}_s^i = \mathcal{P}_{\mathcal{K}}^{(i)}.$$

Remark 5: Note that, as the level of the defectiveness varies, it is intuitive to conjecture that there exists a monotonicity with respect to the relevant system performance (e.g., in this brief, the bound of the stochastic perturbations that the system can tolerate without becoming unstable), which we will verify via the numerical examples in next section.

IV. Numerical Examples

In this section, three examples are presented to verify the theoretical findings. For description brevity, we denote ith row of the rth vertex in the polytope uncertainty description as $\Lambda_r^i, \forall i \in \mathcal{I}$, $\forall r = 1, \ldots, M$.

Example 1: Consider a three-neuron MJNN of four jumping modes with defective TPM (4) to be given by

$$A_1 = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.3 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.7 \end{bmatrix},\ A_4 = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.7 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0.19 & -0.21 & 0.09 \\ 0.00 & -0.31 & 0.19 \\ -0.20 & -0.10 & -0.20 \end{bmatrix}$$
$$B_2 = \begin{bmatrix} 0.21 & -0.20 & 0.10 \\ 0.00 & -0.30 & 0.19 \\ -0.21 & -0.10 & -0.20 \end{bmatrix}$$
$$B_3 = \begin{bmatrix} 0.20 & -0.20 & 0.10 \\ 0.00 & -0.27 & 0.21 \\ -0.21 & -0.12 & -0.19 \end{bmatrix}$$
$$B_4 = \begin{bmatrix} 0.10 & -0.20 & 0.21 \\ 0.10 & -0.20 & 0.12 \\ -0.10 & -0.12 & -0.30 \end{bmatrix}$$
$$M_i = 0.3I,\ N_{1i} = 0.1I,\ N_{2i} = 0.2I,\ i = 1, 2, 3, 4$$
$$h = 0.01,\ \rho = 0.3.$$

The TPM comprises five vertices Λ_r, $r = 1, 2, \ldots, 5$, and their second lines Λ_r^2, $r = 1, 2, \ldots, 5$, are given by

$$\Lambda_1^2 = [\ ?\ \ 0.15\ \ 0.30\ \ ?\ ],\ \Lambda_2^2 = [\ ?\ \ 0.15\ \ 0.60\ \ ?\ ]$$
$$\Lambda_3^2 = [\ ?\ \ 0.45\ \ 0.30\ \ ?\ ],\ \Lambda_4^2 = [\ ?\ \ 0.45\ \ 0.55\ \ ?\ ]$$
$$\Lambda_5^2 = [\ ?\ \ 0.40\ \ 0.60\ \ ?\ ]$$

and other rows in the five vertices are defined with the same elements, $\forall r = 1, 2, \ldots, 5$

$$\Lambda_r^1 = [\ ?\ \ 0.4\ \ ?\ \ 0.2\ ],\ \Lambda_r^3 = [\ ?\ \ 0.2\ \ 0.5\ \ ?\ ]$$
$$\Lambda_r^4 = [\ ?\ \ 0.3\ \ ?\ \ ?\ ].$$

For simplicity, the TPM in the polytope uncertainty description can be rewritten in the following norm-bounded form:

$$\begin{bmatrix} ? & 0.4 & ? & 0.2 \\ ? & [0.15, 0.45] & [0.3, 0.6] & ? \\ ? & 0.2 & 0.5 & ? \\ ? & 0.3 & ? & ? \end{bmatrix}. \quad (23)$$

By Theorem 1, one can verify that (5)–(6) have a feasible solution, which shows that the given system is asymptotically

TABLE I
STABILITY OF THE MJNN CORRESPONDING TO DIFFERENT ρ VALUES

Value of ρ	Stability of MJNN
0.3	Stable
0.4	Stable
0.5	Unstable
0.6	Unstable

TABLE II
MAXIMUM VALUE OF ρ FOR UNCERTAIN TPs WITH DIFFERENT INTERVALS

Interval of π_{22}	Interval of π_{23}	Maximum value of ρ
[0.05, 0.85]	[0.10, 0.90]	0.389
[0.15, 0.75]	[0.20, 0.80]	0.412
[0.25, 0.65]	[0.30, 0.70]	0.439
[0.35, 0.55]	[0.40, 0.60]	0.441

stable in the mean square despite the defectiveness existing in TPM (23).

Note that, as shown in (3), ρ has a constraint on the intensity of stochastic perturbations. This means that a larger ρ may cause the corresponding MJNN to become unstable. By Theorem 1, one can further obtain the relation between the different ρ and the stability of the resulting MJNN, as listed in Table I. It is seen from Table I that a larger ρ, which allows the stochastic perturbations $\sigma(y(k), k)w(k)$ to be more intense, will lead to the instability of the system. Thus a direct question is: what is the factor that gives rise to different maximum values of ρ such that the corresponding MJNN is unstable? It is natural for us to conjecture that different defectiveness of a TPM may have such a potential. That is, as the level of the defectiveness varies, the maximum value of ρ will change. The corresponding verification will be shown in Examples 2 and 3.

Example 2: Consider the MJNN in Example 1 and change the intervals of uncertain TPs π_{22} and π_{23} in (23). The purpose here is to demonstrate the different behaviors of the underlying MJNN as the intervals of uncertain TPs vary. Using the conditions in Theorem 1, we can obtain the maximum value of ρ by solving the following minimization problem:

$$\begin{aligned} &\min\ 1/\rho \\ &\text{subject to } LMIs \text{ (5) and (6).} \end{aligned}$$

Given four different intervals of π_{22} and π_{23}, the corresponding computation results are shown in Table II. It can be seen that, as the intervals of uncertain TPs π_{22} and π_{23} become smaller, the maximum value of ρ increases, i.e., more intense stochastic perturbations are allowed.

Now, we will consider the more complex cases in Example 3, in which all the three types of TPs are involved in the variations.

Example 3: Consider the MJNN in Example 1 with four different defective TPMs as listed in Table III.

From Cases I–IV in Table III, the level of the defectiveness decreases, which one can observe in three cases: 1) unknown elements turn into uncertain or even known ones; 2) the intervals of the uncertain elements become tighter; and 3) the uncertain elements become known ones. In particular, Case I represents the so-called switched NNs under arbitrary switching and Case IV represents the conventional MJNN with completely known TPM. The corresponding result can be seen in Table IV.

TABLE III
FOUR DIFFERENT TRANSITION PROBABILITY MATRICES

Case I: Completely unknown TPM

$$\begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}$$

Case II: Defective TPM 1

$$\begin{bmatrix} ? & 0.4 & ? & 0.2 \\ 0.2 & ? & [0.3, 0.5] & 0.1 \\ ? & 0.2 & [0.1, 0.7] & ? \\ ? & ? & ? & ? \end{bmatrix}$$

Case III: Defective TPM 2

$$\begin{bmatrix} ? & 0.4 & ? & 0.2 \\ 0.2 & [0.15, 0.35] & [0.3, 0.5] & 0.1 \\ ? & 0.2 & [0.4, 0.6] & ? \\ 0.4 & 0.3 & 0.2 & 0.1 \end{bmatrix}$$

Case IV: Completely known TPM

$$\begin{bmatrix} 0.3 & 0.4 & 0.1 & 0.2 \\ 0.2 & 0.3 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.4 & 0.3 & 0.2 & 0.1 \end{bmatrix}$$

TABLE IV
MAXIMUM VALUE OF ρ FOR DIFFERENT CASES

Case	Maximum value of ρ
I	0.134
II	0.296
III	0.438
IV	0.525

From the computation results, it can be also seen that the lower is the level of defectiveness of the TPM, the stronger is the capability of tolerating stochastic perturbations for ensuring stability of the system.

As seen from Example 1, the validity of Theorem 1 is demonstrated. Also, it can be concluded from Examples 2 and 3 that, as more statistics are available to the designers, the relevant system performance (the capability of tolerating stochastic perturbations here) will be improved as conjectured.

V. CONCLUSION

This brief dealt with the stability criterion for a class of uncertain MJNNs with defective statistics of modes transitions in discrete time domain. The defective TPs took account of the recent studies, i.e., the so-called uncertain TPs and partially unknown TPs, in a composite way. By using the property of the TPM and the convexity of uncertain domains, a sufficient condition for the stability of the underlying system was established. Furthermore, a monotonicity between the level of the defectiveness and the system's capability of tolerating the stochastic perturbations was observed concerning the maximum value of a given scalar ρ. Numerical examples were provided to show the effectiveness of the developed results. It is worth mentioning that the consideration of the defective TPM can be further extended to other issues of MJNNs, such as MJNNs with time delays [17], [18], MJNNs in continuous time domain [19], etc.

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