Supplementary Materials of "High-Level Planner Synthesis for Whole-Body Locomotion in Unstructured Environments"

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I. Linear Temporal Logic Semantics

The semantics of LTL are defined inductively as

$$
\begin{aligned}
(q_i, p_i) &\models \neg\varphi \text{ iff } (q_i, p_i) \not\models \varphi \\
(q_i, p_i) &\models \varphi_1 \wedge \varphi_2 \text{ iff } (q_i, p_i) \models \varphi_1 \wedge (q_i, p_i) \models \varphi_2 \\
(q_i, p_i) &\models \varphi_1 \vee \varphi_2 \text{ iff } (q_i, p_i) \models \varphi_1 \vee (q_i, p_i) \models \varphi_2 \\
(q_i, p_i) &\models \bigcirc\varphi \text{ iff } (q_{i+1}, p_{i+1}) \models \varphi \\
(q_i, p_i) &\models \varphi_1 \mathcal{U} \varphi_2 \text{ iff } \exists j \geq i, \text{s.t.} (q_j, p_j) \models \varphi_2 \\
&\qquad \text{and } (q_k, p_k) \models \varphi_1, \forall i \leq k \leq j
\end{aligned}
$$

In these definitions, the notation $\bigcirc\varphi$ represents that φ is true at the next "step" (i.e., next position in the sequence), $\Box\varphi$ represents φ is always true (i.e., true at every position of the sequence), $\Diamond\varphi$ represents that φ is eventually true at some position of the sequence, $\Box\Diamond\varphi$ represents that φ is true infinitely often (i.e., eventually become true starting from any position), and $\Diamond\Box\varphi$ represents that φ is eventually always true (i.e., always becomes true after some point in time in the sequence).

II. An Additional LTL Specification For Keyframe State

If $\bigcirc e = \bigcirc e_{hd}$ (i.e., hugeDownward), the level of degree for next keyframe state increases by one or two units, either from step length or apex velocity. The only exception is when $q = q_{l\text{-}l}$, next step state q can only maintain $q_{l\text{-}l}$.

$$
\begin{aligned}
&\Box\big((q_{s\text{-}s} \wedge \bigcirc e_{hd}) \Rightarrow \bigcirc(q_{n\text{-}s} \vee q_{s\text{-}n} \vee q_{l\text{-}s} \vee q_{s\text{-}l} \vee q_{n\text{-}n})\big) \bigwedge \Box\big((q_{s\text{-}n} \wedge \bigcirc e_{hd}) \Rightarrow \bigcirc(q_{s\text{-}l} \vee q_{n\text{-}n} \vee q_{n\text{-}l})\big) \\
&\quad \ldots \\
&\bigwedge \Box\Big(\big((q_{l\text{-}n} \vee q_{n\text{-}l} \vee q_{l\text{-}l}) \wedge \bigcirc e_{hd}\big) \Rightarrow \bigcirc q_{l\text{-}l}\Big) \bigwedge \Box\Big(\big((q_{\text{swing}} \vee q_{\text{stop}}) \wedge \bigcirc e_{hd}\big) \Rightarrow \bigcirc(q_{s\text{-}n} \vee q_{n\text{-}n} \vee q_{l\text{-}n})\Big)
\end{aligned}
$$

where, if $q = q_{s\text{-}s}$, $\bigcirc q$ increases the level of degree by one, i.e., $q_{s\text{-}n}$ and $q_{n\text{-}s}$, or by two, i.e., $q_{n\text{-}n}, q_{l\text{-}s}$ and $q_{s\text{-}l}$. Special cases are $q_{l\text{-}n}, q_{n\text{-}l}$ and $q_{l\text{-}l}$ where $\bigcirc q = \bigcirc q_{l\text{-}l}$ is the only choice. Unexpected events are those in (S5).