Trajectory Optimization through Uncertain Contact with Stochastic Complementarity

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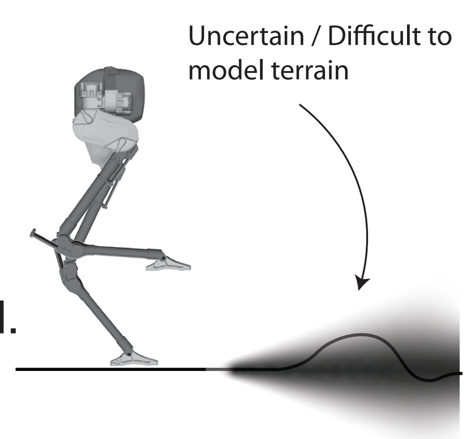
Contact-Sensitive Trajectory Optimization

Designing safe and robust locomotion behaviors for bipedal robots poses a challenge to the dynamic walking field.

Contact-implicit trajectory optimization has recently gained attention for its ability to generate diverse locomotion behaviors [1,2].

The contact-implicit approach solves for contact forces, configuration, and control trajectories simultaneously but requires a model of the terrain.



Solutions are sensitive to errors in the model; errors in the terrain model could cause the robot to slip and fall.

Previous works generate an ensemble of trajectories by perturbing model parameters [3].

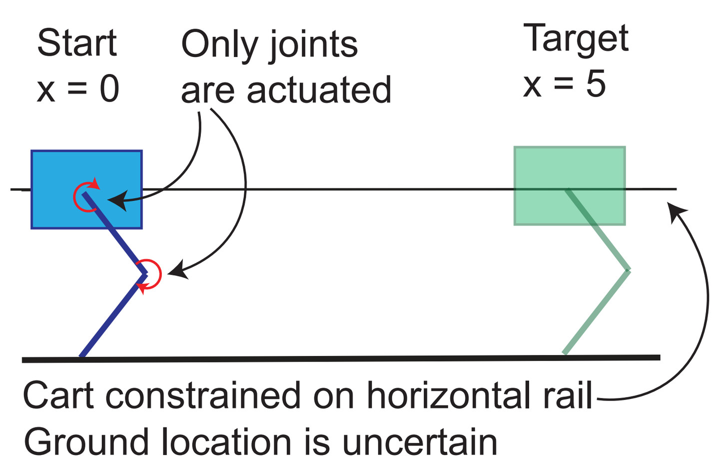
We propose explicitly accounting for parametric uncertainty in the terrain during trajectory optimization.

Objectives

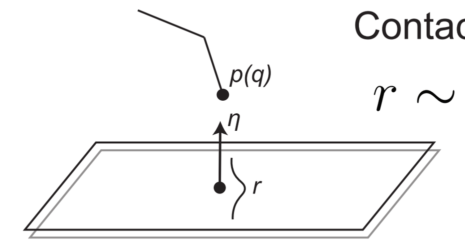
1. **Model parametric uncertainty** in the friction coefficient and contact point location and develop a corresponding contact-sensitive objective.
2. Demonstrate that **trajectories generated under high uncertainty are robust** to changes in the uncertain parameters
3. Demonstrate that **trajectories converge** to the trajectories generated without the contact-sensitive objectives **as the uncertainty vanishes.**

Location Uncertainty Increases Contact Distance

Contact-Driven Cart Problem



Uncertain Contact Point



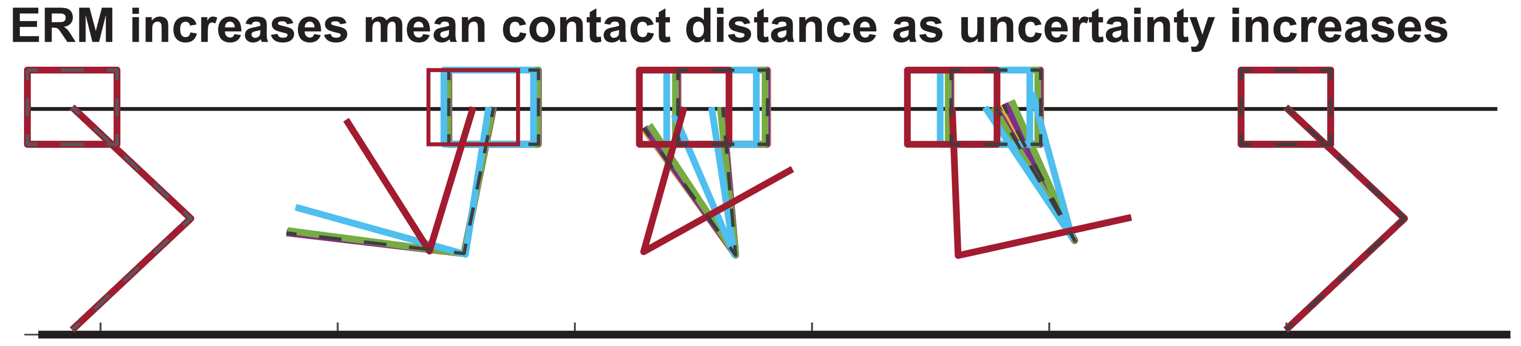
Contact Point Distribution

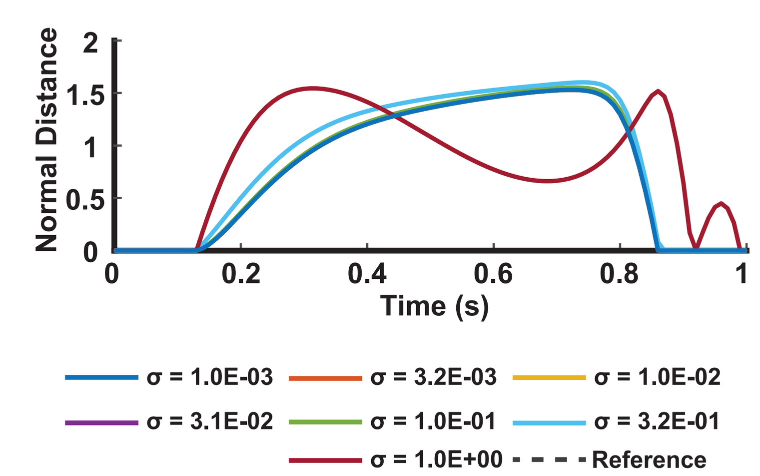
$$r \sim \mathcal{N}(\bar{r}, \Sigma_r)$$

Normal Distance Distribution

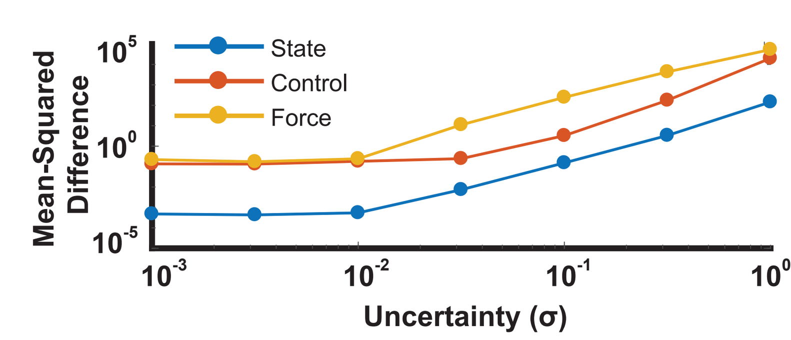
$$\phi(q) = \eta^T(p(q) - r)$$
$$\sim \mathcal{N}(\eta^\top(p(q) - \bar{r}), \eta^\top \Sigma_r \eta)$$

ERM increases mean contact distance as uncertainty increases



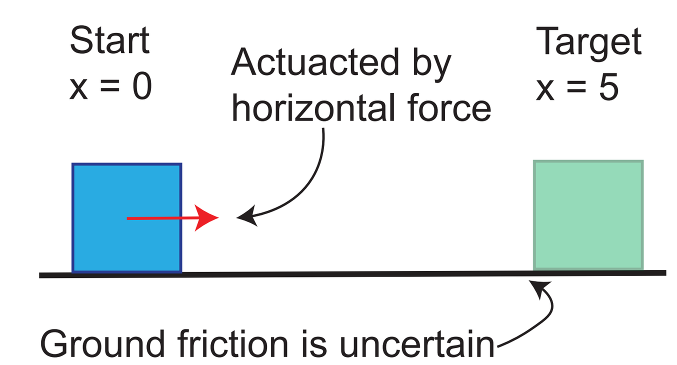


ERM converges to non-robust solution as uncertainty decreases

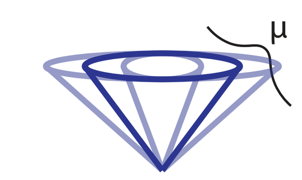


Friction Uncertainty Shortens Sliding Phase

Sliding Block Problem



Uncertain Friction Coefficient



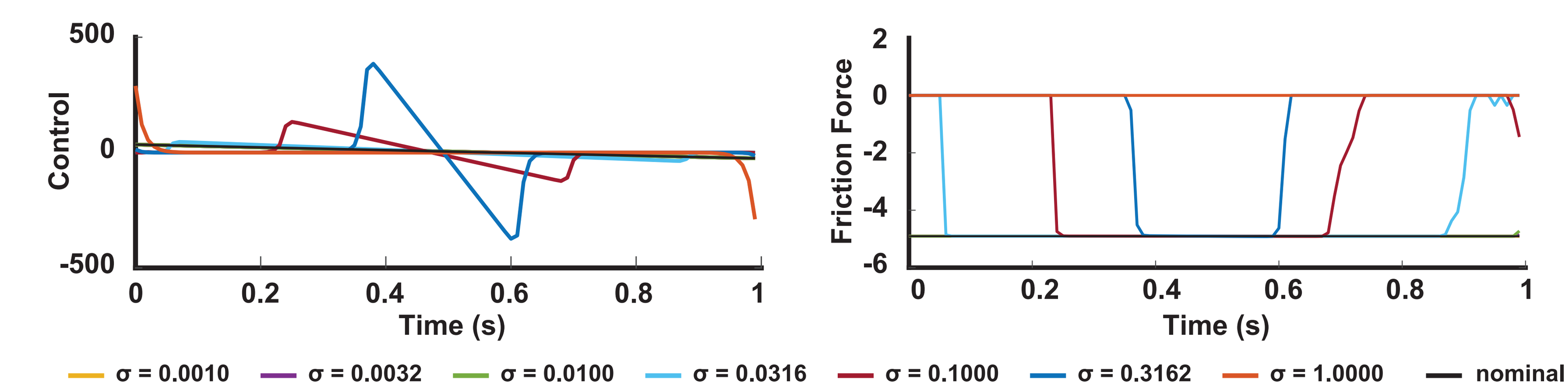
Friction Coefficient Distribution

$$\mu \sim \mathcal{N}(\bar{\mu}, \sigma_\mu)$$

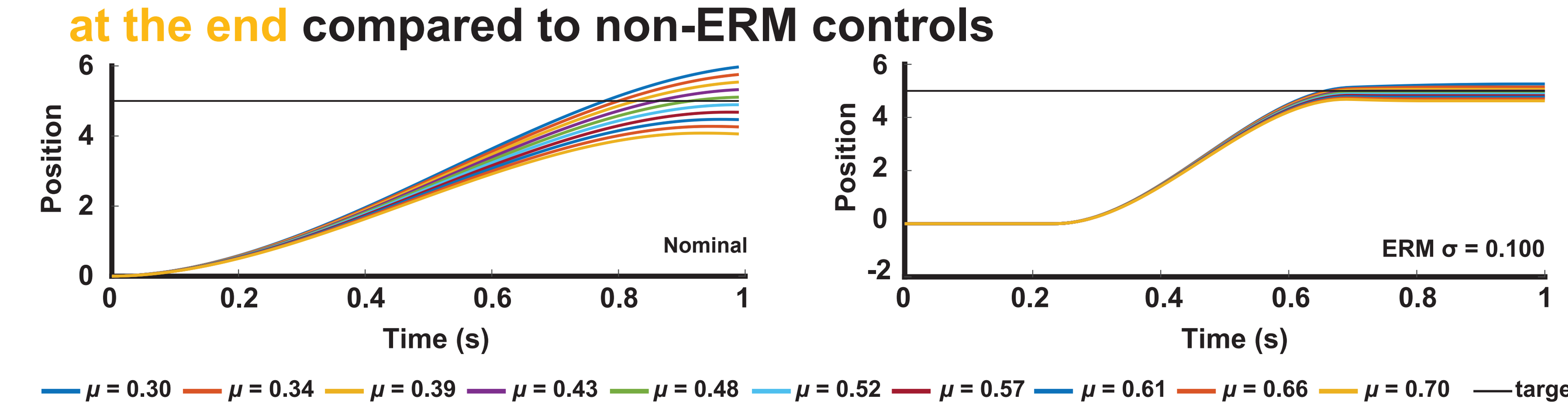
Friction Cone Defect Distribution

$$F(\lambda) = \mu\lambda_N - \lambda_T$$
$$\sim \mathcal{N}(\bar{\mu}\lambda_N - \lambda_T, \sigma_\mu \lambda_N)$$

ERM increases control effort and reduces sliding phase duration



Open loop simulations with **ERM-based controls have less variation at the end** compared to non-ERM controls



Expected Residual Minimization: Formulation and Behavior under Changes in Uncertainty

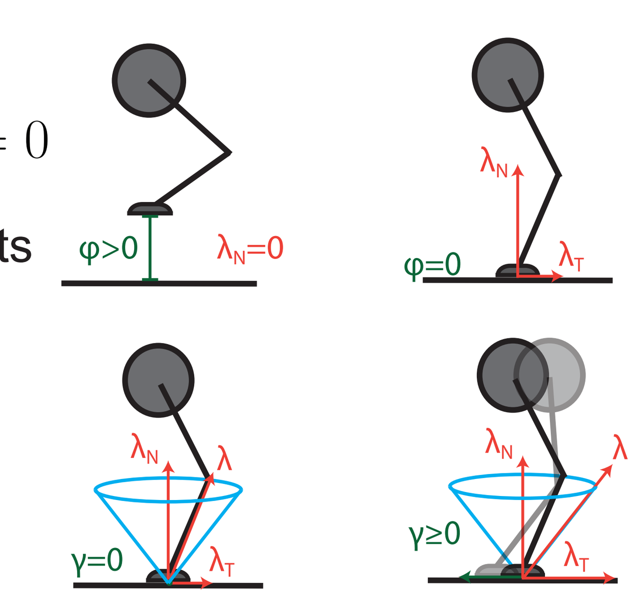
Standard Complementarity Problem for Contact

Normal Distance Constraint

$$\phi(q) \geq 0, \quad \lambda_N \geq 0, \quad \phi(q)^\top \lambda_N = 0$$

Friction Cone and Sliding Constraints

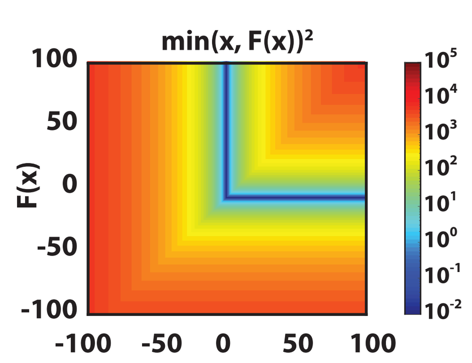
$$\lambda_T \geq 0, \quad \gamma + J_T^\top \dot{q} \geq 0,$$
$$\lambda_T^\top(\gamma + J_T^\top \dot{q}) = 0$$
$$\gamma \geq 0, \quad \mu\lambda_N - \lambda_T \geq 0,$$
$$\gamma^\top(\mu\lambda_N - \lambda_T) = 0$$



Expected Residual Minimization (ERM) for Complementarity with Uncertain Parameters

Replace complementarity constraint with residual

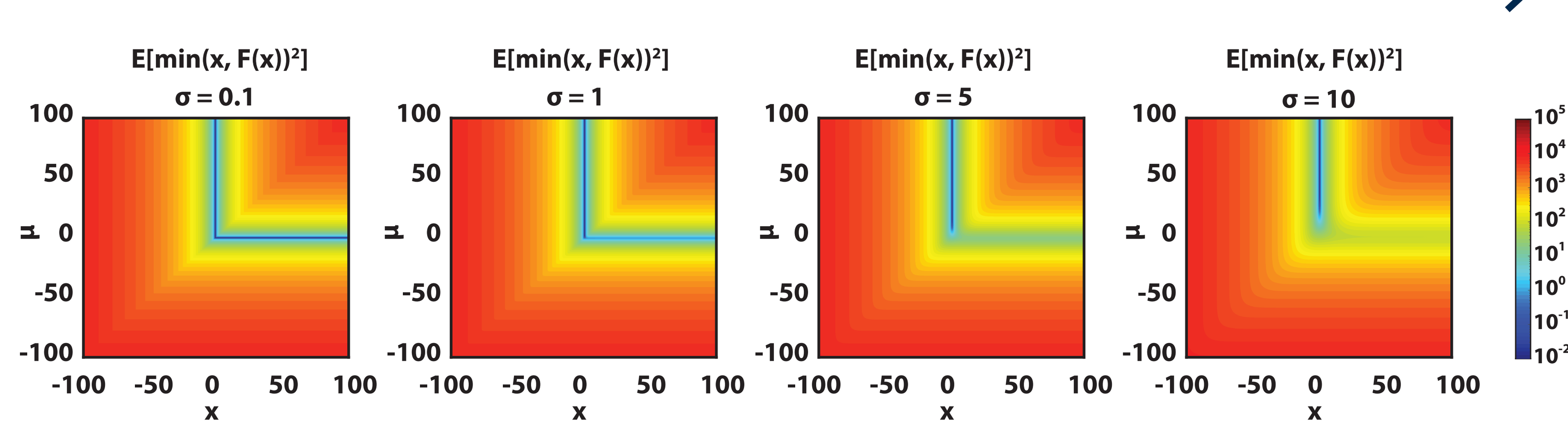
$$\min(x, F(x)) = 0 \iff x \geq 0, F(x) \geq 0, x^\top F(x) = 0$$



Assume one of the variables is normally distributed. The Expected Residual [4]:

$$F(x) \sim \mathcal{N}(\mu, \sigma)$$
$$\mathbb{E}[\min(x, F(x))^2] = x^2 - \sigma^2(x - \mu)p(x) + (\sigma^2 + \mu^2 - x^2)P(x)$$

Increasing uncertainty leads to increasing mean uncertain variables



Decreasing uncertainty leads to the complementarity solution

Proposition: As uncertainty decreases, the **ERM cost function converges to the complementarity residual function** with the mean of the uncertain variable. That is:

$$\lim_{\sigma \to 0^+} \mathbb{E}[\min(x, F(x))^2] = \min(x, \mu)^2$$

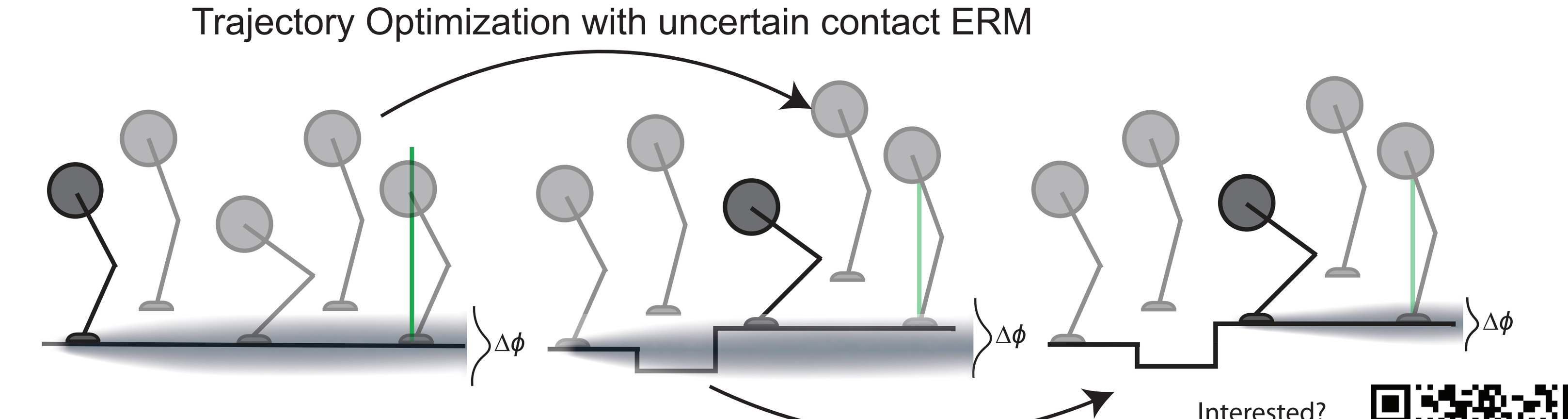
where μ is the mean value of the uncertain constraint F(x)

Discussion and Future Works

We investigated the use of the ERM method [4] to model uncertainty in the terrain parameters and generate robust trajectories.

1. **Modeled uncertainty** in the friction coefficient and in the contact location.
2. Demonstrated the **ERM method generates trajectories which are robust** to uncertainty in terrain parameters
3. Demonstrated that the **ERM-generated trajectories approach non-ERM trajectories** as the uncertainty decreases.

Future work could combine ERM with Bayesian inference to estimate terrain parameters in ERM from locomotion data.



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References: [1] M. Posa, C. Cantu, and R. Tedrake. (2014). [2]. A. Patel, et al. (2019). [3]. I. Mordatch, K. Lowrey, and E. Todorov. (2015). [4]. Y. Tassa and E. Todorov (2010).