Mass Estimation of a Moving Object Through Minimal Manipulation Interaction

Sergio Aguilera¹, Muhammad Ali Murtaza¹, Ye Zhao¹ and Seth Hutchinson¹

Abstract—In this paper, we study the problem of dynamic interaction between a robot and an unknown object (e.g., catching a ball, or handing off an object during locomotion). In particular, we propose a method for estimating the inertial parameters of an object during dynamic interaction, while minimally altering the trajectory of the object — a minimal interaction approach. Our method combines trajectory estimation (e.g., using standard methods from computer vision) with a model-based estimator that exploits the robot’s known dynamic model. We first develop the method for a generalized three-dimensional problem, and then evaluate the method for the case of an object moving along a linear trajectory. We present experimental results obtained using a KUKA iiwa 7 interacting with rolling balls of varying mass. Our experiments demonstrate that the mass of the objects can be accurately estimated at the moment of impact when accurate object trajectory estimates are available, and that significant improvement can be obtained by incorporating force measurements at the contact point while following the object.

I. INTRODUCTION

The use of lightweight manipulators has gained increasing attention due to their low power consumption and ability to be integrated with a mobile base for mobile manipulation tasks. As these manipulators are deployed to unpredictable environments, they will interact with unknown objects which may be moving or even thrown to the robot. When studying the interaction with a moving object, most past research has focused on catching the object itself by designing algorithms to generate catching trajectories and/or improving the grasping. However, before grasping a moving object, we should ask if the manipulator will be able to handle this interaction, e.g., will the manipulator motors provide sufficient torque to stop the object’s motion. For this end, we propose a method to estimate the mass of a moving object before grasping by performing a minimal interaction between the end effector and the object to estimate its mass.

Manipulation of a moving object such as catching a ball [12], [16], [10], juggling [4] or throwing [21] has been widely studied since the early 2000s. Specifically, object catching midair research has mainly focused on the grasping task itself. Early research focused on the static catching, aiming to put the end-effector at the desired position under the time constraint to catch the object [5], [11], [1]. Later, studies on low impact [12] and soft catching [7], match the velocity of the object with the end effector to accomplish a smoother interaction with the object, increasing the catching time for the hand. Others have focused on how to catch irregular shaped objects [3], taking advantage of the shape of the object to come up with novel catching techniques [10], [17], or using highly dexterous grasping hands [16]. Regardless of their main objectives, all the previous work adopts a similar pipeline for the catching procedure: i) visually detect the object, ii) compute the interception point, iii) solve inverse kinematics of the manipulator to move the end effector to the desired position. Furthermore, objects that are being caught often have a light weight, usually including softballs, empty/partially filled bottles, or elements with complex shapes but little mass. This allows to tackle the catching problem from an inverse kinematics perspective since there are no worries about the torques that the manipulator might have to apply at each joint to accomplish the task.

When the interaction dynamics are non-negligible, e.g., catching a heavy object, it is imperative to estimate the inertia parameters of the objects as the interaction occurs. Research on estimation of inertial properties is extensive, as shown in [6] with three different approaches: purely visual, exploratory, and fixed-object. Visual methods rely on the estimation of the object geometry and depend on a number of assumptions of the object’s properties to estimate the mass and 3D inertial parameters. Fixed-object methods rely on known manipulator dynamics, and by moving the arm around, are able to estimate 3D inertial parameters accurately. Exploratory methods perform basic interactions, such as pushing or tilting, to a static object place on a surface, to acquire an accurate estimation of 2D and 3D inertial parameters [19], [20], [13], [18], [8]. This class of methods heavily relies on the estimation of the static and dynamic friction coefficients between the object and the surface.

In this paper, we propose an exploratory method that will estimate the mass of an object in motion, such as falling or rolling down a slope. In the moving object case we can neglect the estimation of the friction coefficients and directly estimate the object’s mass. Nevertheless, the estimation needs to be accomplished promptly, while the object is still inside the reachable manipulator task space. In particular, we propose a minimal interaction method for mass estimation of a moving object. By minimal interaction, we refer to an interaction that seeks to modify as little as possible the object default position and velocity trajectories while attempting to accomplish a specific goal, in our case, the estimation of mass. To accomplish this, we leverage the general object catching framework. Starting by estimating the trajectory of the object, we compute an interception point in the task space of the manipulator. When the object arrives at the interception point, the end effector will be moving

¹Institute of Robotics and Intelligent Machines, Georgia Institute of Technology, Atlanta, GA 30332, USA sfaguile@gatech.edu
at the same speed along the estimated object trajectory. Then, the end-effector motion will slow down so that the object starts to interact with the manipulator. Using the measured force on the end effector, we will estimate the mass of the object. By modulating the velocity of the end effector, the contact force can be adjusted to avoid motor over-actuation while accomplishing the estimation objective. The main contributions of this work are:

- A general framework for minimal interaction, aiming to interact with a moving object while minimizing the changes of the object’s trajectory.
- A rapid exploratory method for mass estimation of moving objects. Our method allows to obtain an estimate of the mass without friction coefficient information.
- Experimental validation of our methodology. We show that the method is able to estimate a variety of object masses.

This paper is organized as follows. Section II introduces the minimal interaction framework and presents the method of object mass estimation. The experiments with spheres rolling down a slope are shown in Section III. Finally, we conclude in Section IV.

II. METHODOLOGY

A. Minimal Interaction

Few studies have analyzed the interaction forces between the end-effector and object, while minimizing the force sensed by the object [15] and making the manipulator react to unknown external forces due to the interaction with an object [2]. Our minimal interaction approach generates a controlled interaction between the manipulator and a moving object to obtain a mass estimation while modulating the object’s trajectory and velocity to a minimal extent.

Consider a rigid body that will interact with the manipulator as shown in Fig. 1. Define the inertial frame as $F_0$ and the body frame as $F_0^b$ at the body’s center of mass (CoM). The object is moving in three-dimensional space with linear and angular velocities $v^a(t)$ and $\omega^a(t)$, respectively. The rigid motion of the body in 3D-space can be described as in [9],

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{v}^a \\ \dot{\omega}^a \end{bmatrix} + \begin{bmatrix} \omega^a \times & 0 \\ 0 & \omega^a \times \end{bmatrix} \begin{bmatrix} mI & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} v^a \\ \omega^a \end{bmatrix} = \begin{bmatrix} f^a \\ \tau^a \end{bmatrix}$$

(1)

where $m$ is the mass and $I$ is the inertia of the body at the CoM of the body frame; $V^a, V^a \in \mathbb{R}^6$ are the body velocity and acceleration, respectively; $\dot{v}^a$ and $\dot{\omega}^a$ are the linear and angular accelerations in the body frame; $\omega^a \times$ is the skew-symmetric matrix of $\omega^a$; and $F^a$ and $\tau^a$ are the forces and torques applied onto the body, through the CoM.

As the object moves in the inertial frame we can define the trajectory of the CoM relative to the inertial frame, parameterized by time, as $p(t) \in SE(3)$. At any point of the trajectory, the object will have a spatial velocity

$$V^0(t) = \begin{bmatrix} v^0(t) \\ \omega^0(t) \end{bmatrix}$$

Fig. 1. A free-floating rigid body moving along a parabolic trajectory subject to gravity only.

where $\omega^0$ is tangent to the trajectory of motion.

The framework that we propose is analogous to the catching maneuvers:

1) Trajectory prediction: Using image processing on the moving object, predict the trajectory $\hat{p}(t)$ and spatial velocity $\hat{V}^0(t)$ of the body over time.
2) Compute the interception with manipulator’s task-space: Given the predicted body trajectory, check if it goes through the manipulator task-space for a considerable time, since our interaction needs to interact with the object at least for 300[ms].
3) Compute the interception point: Given the position and velocity of the object, compute the time until the object enters the task space. Given the manipulator’s configuration $(q)$, select an interception point $p^*$ on the object’s trajectory, which the end effector can reach before the object arrives.
4) Set end-effector position and velocity: Before coming into contact with the object, set the end effector to move along the predicted trajectory at a given linear velocity $v^0_{\text{ref}} < \hat{v}^0$.
5) Collide with the object: Measure the contact force between the object and the end-effector to estimate the mass of the object $\hat{m}$
6) Track the contact: Maintain the end effector on the estimated trajectory (which is modified by the interaction) while maintaining the contact with the object to improve the estimation of mass.

It is important to notice that even though we are tracking the position and velocity of the object’s CoM, the contact is made with a point on the surface of the object. To obtain the desired interaction, the manipulator will interact with a point on the frontal surface of the object. If the object is rotating there will be a friction between the end effector, but for this study we will consider it to be negligible.

To obtain the force applied to the object by the end effector of the manipulator, we consider the general dynamic equation for an $n$-DoF manipulator given by

$$M(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau + \tau_c$$

(2)

where $q, \dot{q}$ and $\ddot{q} \in \mathbb{R}^n$ represent joint position, velocity and acceleration, respectively; $M(q) \in \mathbb{R}^{n\times n}$ is the symmetric positive definite inertia matrix; $C(q, \dot{q}) \in \mathbb{R}^n$ is the Coriolis and Centrifugal vector; $G(q) \in \mathbb{R}^n$ is the gravity vector; $\tau$,
\( \tau_c \in \mathbb{R}^n \) denote the commanded joint torque and joint torque associated to a generalized external torque respectively, such that
\[
\tau_c = (J(q)^T) F_{ee}^0
\]
where \( F_{ee}^0 \in \mathbb{R}^6 \) represents the spatial wrench applied to the end-effector. \( J(q)^T \in \mathbb{R}^{6 \times n} \) is the spatial Jacobian of the manipulator which maps the joint velocities \( \dot{q} \) to the end-effector spatial velocity \( \dot{V}_{ee}^0 \).

**B. Mass Estimation**

For mass estimation, we will focus the analysis on the object. For this, we will assume the angular velocity of the object \( \omega_a \) is small. If the angular velocity of the body is large, at the moment of contact with the end effector, the torque produced due to the friction between the object and the end effector would drastically change the trajectory of the object. Also, if the object has an irregular shape, it would be impossible to keep a static contact with the object. Thus, in this paper for mass estimation we will consider the linear part of the rigid body dynamics of Eq. 1 and neglect the effects of the angular velocity on the body.

Before contact, we enforce that the end effector is moving at a speed \( v_{ee}^0 \) and we have an estimation of the linear velocity of the object \( \hat{v}_a^0 \) and an estimation of the rotation of the body frame \( R_{0a} \) relative to the spatial frame.

To make the contact with the object, the end effector moves on the predicted trajectory and \( \hat{v}_a^0 = v_{ee}^0 + \delta \) with an increment \( \delta \in \mathbb{R}^3 > 0 \). We can modify the value of \( \delta \), by modulating the velocity \( v_{ee}^0 \). The value of \( \delta \) will define our initial peak force and if too large the object will not bounce off the end-effector. After contact, \( \hat{v}_a^0 = v_{ee}^0 \) and the estimation of \( \hat{v}_a \) is no longer needed.

Let us inspect the free body diagram from the body's perspective as seen in Fig. 2 when the body comes in contact with the end effector. First, we need to change the reference frame of the external wrenches to the body frame. Since only the linear part of the dynamic equation is considered, we have that
\[
f^a = R_{a0}^T f_{ee}^0 + R_{a0}^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + R_{a0}^T f_{ext}^0
\]
where we have a reactive force \( f_{ee}^0 \), the gravity component and other external forces applied to the object \( f_{ext}^0 \). \( R_{a0}^T \) is the rotation matrix from the inertial coordinate to the body coordinate. As to the dynamics of the CoM, the body velocity will be given by
\[
v = R_{a0}^T v^0
\]
and from the linear part of Eq. 1, assuming the the angular velocity is small
\[
mI \dot{\omega}^a = f^a
\]
Since our goal is to estimate the mass \( m \), we can rearrange the equation, such that
\[
r(m) = mI \dot{\omega}^a - f^a
\]
and the estimation of mass \( \hat{m} \), will be given by minimizing the residual \( r(m) \)
\[
\min_m \ mI \dot{\omega}^a - f^a \\
\text{s.t.} \quad m > 0
\]

We will take a closer look to the variables on Eq. 6. Starting with the acceleration of the body \( \dot{v}^a \) we have three phases of interest: before contact, at the moment of contact, and during the interaction with the manipulator. Before the moment of contact, we have a visual estimate of the velocity \( \dot{v}_0^0 \). The model used could be a ballistic model if the object is being thrown or a linear model if the object is rolling down a slope. At the moment of contact, the object experiences an acceleration
\[
\dot{v}_a^0 = \frac{R_{a0}^T (v^0 - v_{ee}^0)}{\Delta t}
\]
where \( \Delta t \) is the time that it takes to the object to change in velocity. Since we can only get a visual estimation of \( v^0 \) as \( \dot{v}_a^0 \), we can use Eq. 8 to have an estimate of the acceleration \( \dot{v}_a^0 \), which can be used in Eq. 9 to obtain a first estimation of mass. For the case presented in Fig. 2, we will have
\[
\min_m \ mI \dot{\omega}^a - R_{a0}^T f_{ee}^0 - R_{a0}^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \\
\text{s.t.} \quad m > 0
\]
where \( f_{ee}^0 \) is measured by the end effector and \( R_{a0}^T \) is visually estimated.

After the initial contact, the object moves with the end-effector and estimation of the velocity is not needed since they are moving together, and \( \dot{v}_a^0 \) becomes a control variable. Now, we examine the force \( f^a \) defined in Eq. 4. The first term \( f_{ee}^0 \) represents the measured force at the contact, the second one due to gravity, and the third due to external forces \( f_{ext}^0 \). In this work we are studying objects that freely move either falling in which case \( f_{ext}^0 = 0 \) or rolling down a slope, with \( f_{ext}^0 = N \), the normal force exerted on the object from the contact surface. When free falling, the object follows a parabolic trajectory and we define a frame \( \mathcal{F}_p \) attached to the body, with the \( x \) direction tangential to the trajectory. Since the end effector moves along with the trajectory of the object, the force applied by the end effector is also tangential.
to the trajectory as shown in Fig. 2, which gives as that in the direction of motion $\tau^p_x$,

$$f_{ee}^p = mg \sin \theta - m \dot{v}_x^p$$

and in particular, the mass estimation will be

$$m = \frac{f_{ee}^p}{g \sin \theta - \dot{v}_x^p}$$  \hspace{1cm} (10)

By changing the velocity of the end effector we can modulate the contact force as shown in Fig. 3. There are three regions of interest:

i) $\dot{v}_x^p > g \sin(\theta)$: the end effector accelerates faster than the object, and thus they will decouple and stop the interaction.

ii) $\dot{v}_x^p < g \sin(\theta)$: there is an interaction between the object and the end effector and the object will move at the speed of the end effector.

iii) $\dot{v}_x^p = g \sin(\theta)$: When the acceleration of the manipulator is equal to that of the object, then we have $f_{ee}^0 = 0$. Even though they might be touching, there is no interaction force.

![Fig. 3. Contact force as a function of $\dot{v}_x$.](image)

As we estimate the mass, the denominator of Eq. 10 approaches zero as the end effector’s acceleration approaches the natural acceleration of the object but the measured force also approaches zero,

$$\lim_{\dot{v}_x \rightarrow g \sin(\theta)} f_{ee}^0 = 0.$$ \hspace{1cm} (11)

Since noise exists in the contact force measurement, we should aim to avoid approaching the natural acceleration of the object, because if the measurement noise is close in magnitude to the measured force, the mass estimation will not be reliable.

To address this issue, we can change the contact force as shown in Fig. 4 by modulating the acceleration of the end effector. If the end effector moves at a constant velocity, we can improve our mass estimation since we only have the error associated to the rotation $\dot{R}_{ee0}$ and the noise from the measured contact force. Thus, this estimation will be more consistent compared to the one made from the initial impact. If we assume the noise from the force measurement to be Gaussian with a zero mean, at each force measurement, we will be able to improve our mass estimation by taking the average over a window of measurements. Since we are considering a catching maneuver, we should consider a window around $100 - 300$ ms. Furthermore, if the force measurement is too small such that signal-to-noise ratio (SNR) is large, we can decelerate the end effector to increase the contact force. Or if the object momentum is too high, the manipulator will have to apply a large force to stay on the trajectory. But we can accelerate the end effector to decrease that interaction force.

![Fig. 4. Interaction force profile.](image)

**III. EXPERIMENT**

For the experimental setup, we will focus on the accuracy of the mass estimation. The manipulator that we use is a Kuka LBR iiwa 7 R800, a lightweight manipulator which can handle up to 7 kg payload. We will have objects rolling down a slope and the manipulator’s end effector will move along the rails in a linear trajectory. The manipulator generate joint torques, $\tau + \tau_c$. With $\tau_c$ and the Jacobian, we can compute the external force at the contact. The communication between the manipulator and the computer is done using Lightweight Communications and Marshaling (LCM) in C++ using the drake toolbox [14]. The measured states of the robot include joint positions and applied torques, and the sent commands are published every 5 ms.

For a rolling ball, we assume no slipping, thus the only difference to Eq. 4 is that we have an external force being applied at the bottom of the ball equal to $f_{ext} = I \dot{v}_x/r^2$ and the Eq. 5 in the direction of object motion is given by

$$m \ddot{v}_x = mg \sin(\theta) - \frac{I \dot{v}_x}{r^2}$$  \hspace{1cm} (12)

and considering that we have a solid sphere, the inertia is given by $I = \frac{2}{5} mr^2$ and the acceleration of the object is given by

$$\dot{v}_x = \frac{5}{7} g \sin(\theta)$$

and the assumption about friction coefficients still holds. Assume that the rail slope $\theta$ is measurable. Thus the only sources of noise come from joint torque sensors and nonlinear geometry deformation from the tool that interact with the object which might deform under large forces. The setup is shown in Fig. 5. Along the trajectory of motion, we always keep the orientation of the end effector, perpendicular to the plane of object motion. Position control is used to control the manipulator to move along the slope at a desired speed. The joint torques are computed via PD impedance control. Thus, if we execute the desired trajectory without any interaction
Moving the end effector at a constant speed, we measured the mass of a solid sphere $m_1 = 4.535$ kg and of a shell-like sphere of $m_2 = 0.433$ kg. Two different slopes were tested $\theta_1 = 25.5^\circ$ and $\theta_2 = 13.49^\circ$. The result for the mass estimation for the heavy ball are shown in Fig. 8 and for the light ball in Fig. 9. These graphs show the estimation of the mass at each time step for an average of the last 5 measurements, which smooth some noise and translate to an interaction after 25 ms. During the initial phase, the balls have not made contact with the end-effector, and we assume that the interaction begins after the first spike. From that moment on we initiate the mass estimation.

A summary of the experiments is presented in Table I. For the light weight ball, the instantaneous error is approximately 33 g, which is consistent with what we expect from our sensor after checking the noise in the measured torque on the unloaded trajectory. Since the ball is light, the tool is stiff enough that it does not deform by the impact. As for the heavy ball, the instantaneous error becomes larger, between $66 - 105$ g, due to the sensor’s error and the deformation of the end-effector. The end-effector bends due to the interaction with the heavy ball, this creates a small oscillatory effect. Regardless of the ball and the slope, we have a high standard deviation of the instantaneous error, but our instantaneous window of 25 ms is also small. If we study the catching task, the interaction duration is still at the order
of magnitude of a few hundred milliseconds, so if we can take a moving average over 200 – 300 ms, we would be able to diminish this noise.

B. Estimation from initial contact

The estimation of mass from the initial contact can be highly helpful as we start to interact with the object and can initialize our estimation as we collect more measurements. As before, the factors that induce estimation errors are not only the torque measurement, and the end-effector tool, but also the estimation of the object velocity before contact as mentioned in Eq. 8. Furthermore, since the torque measurement is going to generate a big spike, the noise from the torque measurement becomes almost insignificant. Depending on the mass of the object, the end-effector tool will act like a damper, as the change in velocity won’t be instantaneous but might take 30 – 80 ms. Thus, the largest error is the estimation of the velocity of the object. For this experiment we will show the computed force by the end-effector onto the object at the initial contact and we have recorded the ball motion to estimate the ball velocity.

We made the experiment with both the heavy and the light balls. The measured force at the initial contact is shown in Fig. 10, while the interaction information and mass estimation is presented in Table II. To estimate the object velocity, we recorded the ball rolling with a 30 fps camera, with the zero initial velocity, until the moment of contact. Using the video, we computed the time between the start of motion of the object until the collision instant and used Eq. 12 to estimate the object velocity. To use Eq. 8 we also need to estimate $\Delta t$ which is the time that it take to the object to acquire the velocity of the end-effector, which was determined to be $\Delta t \approx 50$ ms.

We observe that if the estimation of the object velocity at the impact is accurate, we are able to guarantee a high-fidelity measurement of the mass of the object. Compared to the estimation during contact, we obtain a slightly worse estimations. But if we do a sensitivity analysis to the estimation of mass by adding noise to $\dot{v}^0$, we obtain that the mass estimation error rapidly increases. For example, in the heavy ball case, a 5% error in the velocity (corresponding to a 0.022 m/s difference) increases the estimation error up to 11.5%. Thus, consideration must be taken when evaluating initial mass estimation in reference to the confidence on the velocity estimation of the object that our system has.

IV. CONCLUSION

In this study, we presented an exploratory method to estimate the mass of an object that is moving as a free-floating body or in a rolling down case. We presented a general three-dimensional framework to perform mass estimation and experimentally showed that is able to obtain an accurate estimation in a relative short window of time. This is important as manipulators start to interact with unknown objects in the environment, and being able to quickly estimate parameters enables the robot to plan for future tasks rapidly. Our experiments focused on the estimation of mass, for both light and heavy objects, through a predefined trajectory. This allowed us to confirm the theoretical framework around mass estimation of a moving object. The experimental results also showed the assumption made are appropriate, since the error achieved in the estimation was given by our own sensors and if better sensors are use, a better estimation can be accomplished. We are currently working on the estimation of inertia parameters of the objects in motion through minimal interaction. This is a challenge compared to previous methods which interact with the object in various directions, something that is not possible in our time frame.
REFERENCES


